An Introduction to

Existential Graphs

(a diagrammatic first-order logic)

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Brussels — November 2011

Spoiler Alert / Warning

- take look at logics from broader perspective
  ➪ “philosophical touch” unavoidable

- suppose basic knowledge on first-order predicate calculus:
  $\wedge, \vee, \rightarrow, \exists, \forall, \ldots$; Tarski-style semantics; natural deduction

- easy to grasp formalism (proof by audience participation)

- formal logic can be “intuitive” and fun ☺

Agenda

- philosophical/historical background
- Existential Graphs (EG)
  - basic notation and intuition
  - endoporeutic/game-based semantics
  - diagrammatic inference rules
- retracing classical results of FOPC
- conclusion & back to formal methods. . .

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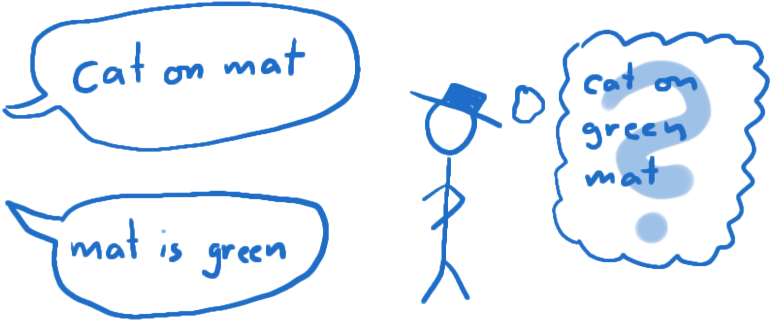
lies on


on (cat, mat)



cat on mat
mat is green
cat on green mat

Point of Departure

Question 1

How to (formally) represent [scientific[1]] knowledge ?

[1]including descriptions of domain experts like software engineers

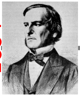
Question 2

How to reason "correctly"[2] in this formal notational system ?

[2]such that formal deduction is not against "intuition"

Historical Background

1854



Bool: propositional reasoning can be based on algebra

Frege: reasoning on arithmetic demands "unalgebraic" notation

"the following holds: there isn't any x and any y such that, if x is a *cat*, then x is not *on* y when y is a *mat*"

$\neg\forall x : \forall y : cat(x) \to (mat(y) \to \neg on(x, y))$

Peirce: general algebra of relations $\Sigma_x \Sigma_y \, (cat_x \cdot mat_y \cdot on_{xy})$

Peano: avoid arithmetic symbols $\exists x : \exists y : cat(x) \wedge mat(y) \wedge on(x, y)$

Peirce: introduces existential graphs

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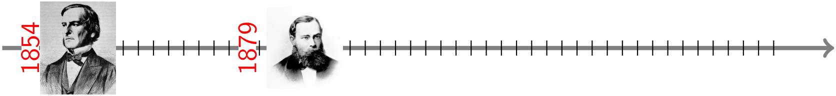

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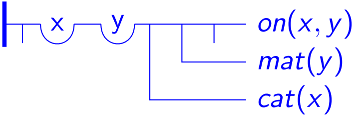

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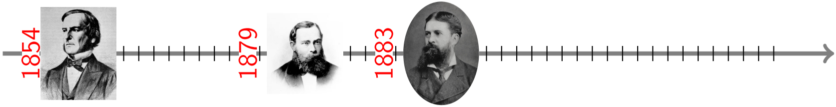

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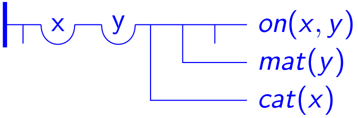

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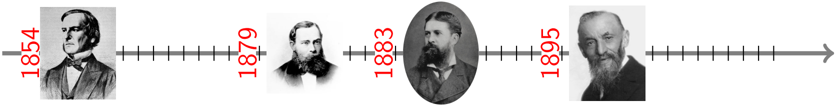

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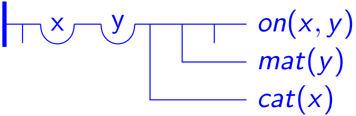

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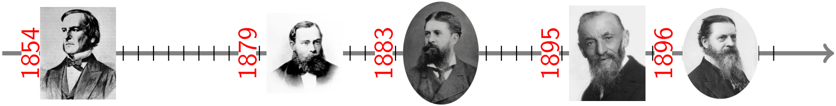

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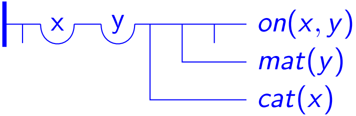

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Peirce's Existential Graphs

I do not think I ever reflect in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose

MS 619: 8, 1909, Studies in Meaning. The Import of Thought

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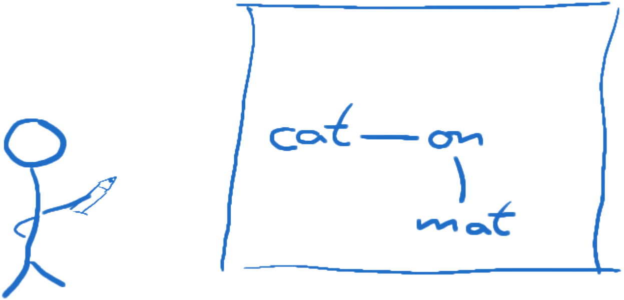

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Existential Graphs : Basics

- sheet of assertion ("$\top$")
- atoms ("n-ary relations")
- line of identity ("$\exists$","$=$")
- juxtapostion ("$\wedge$")
- cut ("$\neg$")
- cuts can be nested

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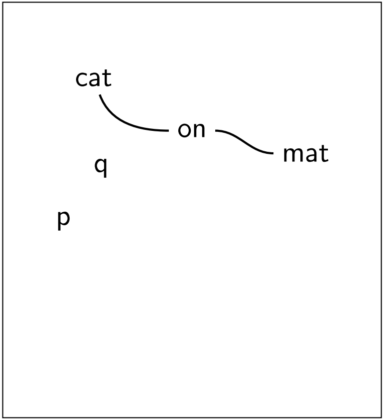

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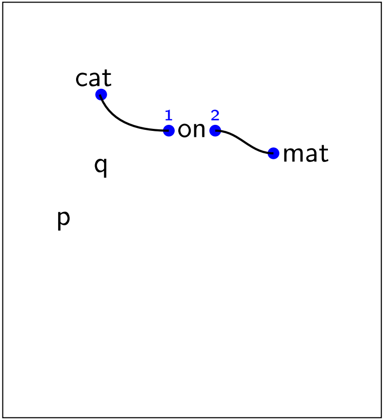

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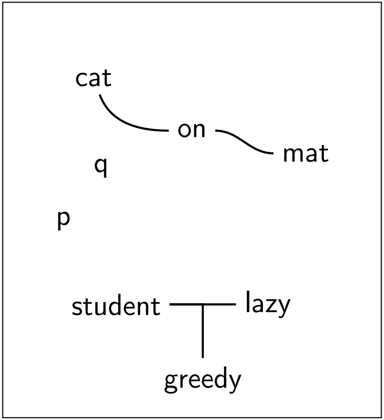

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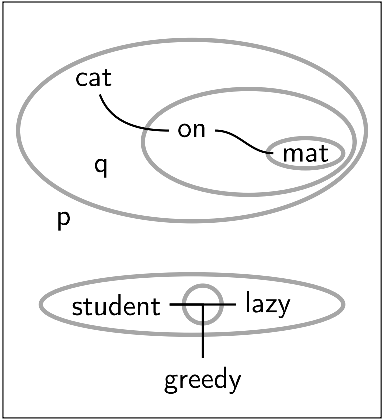

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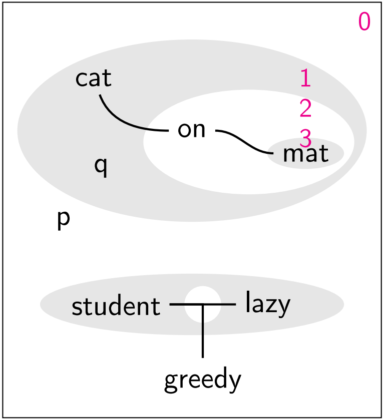

Examples

1. ——— student

2. ——— student

3. ————— student

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1. —— student $\exists x : student(x)$

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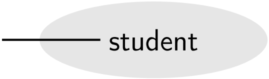
3. 

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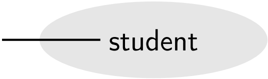
3. 

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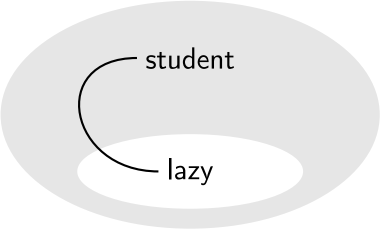
More Examples



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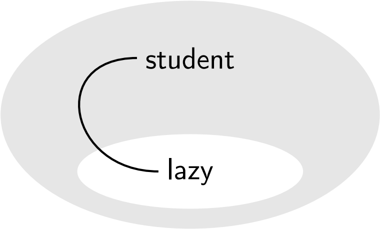

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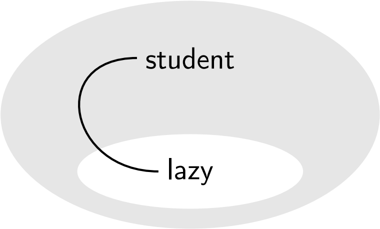

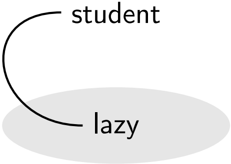

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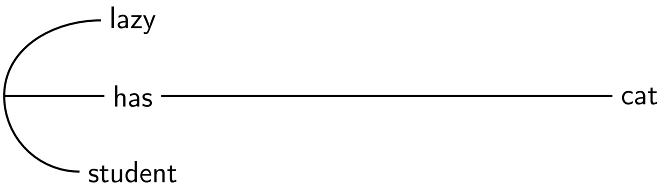

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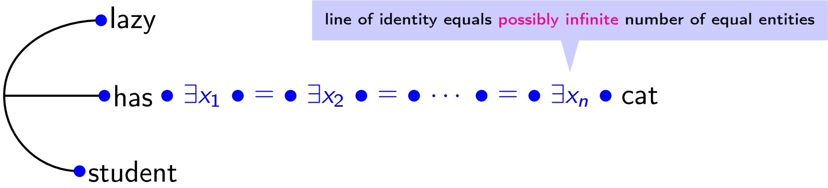

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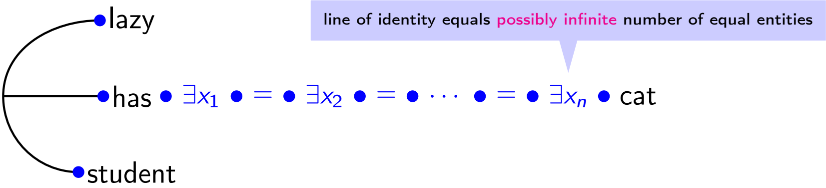

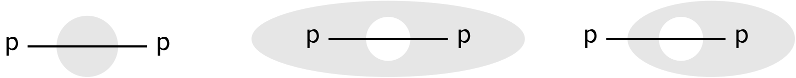

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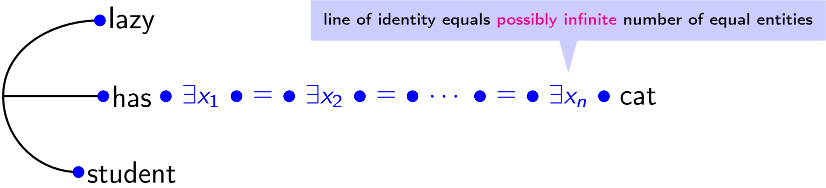

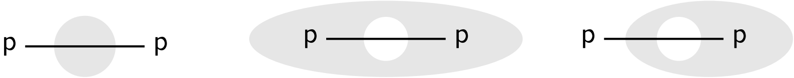

$\exists x : \exists y : p(x) \wedge p(y) \wedge \neg(x = y)$

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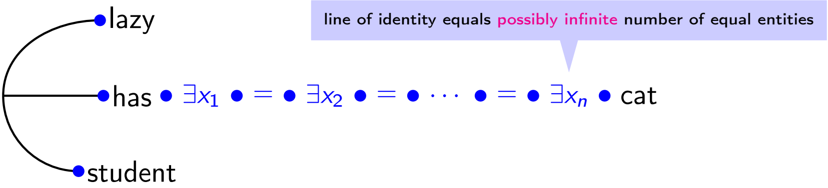

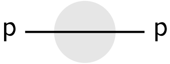

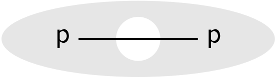

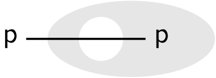

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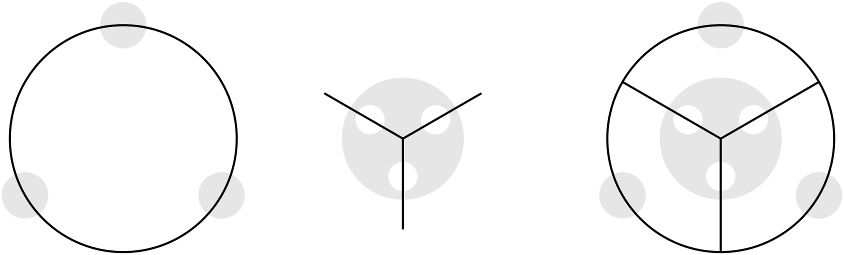
Enough Examples !?



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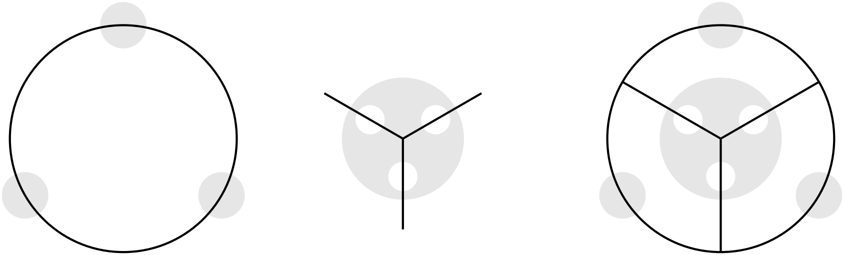

$\exists x, y, z : x \neq y \land y \neq z \land z \neq x$

there are at least three different things

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$\exists x, y, z : \neg \exists w : w \neq x \wedge y \neq x \wedge w \neq z$

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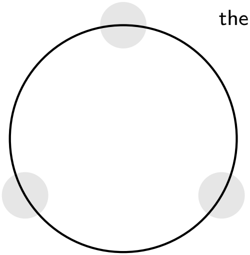

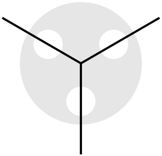

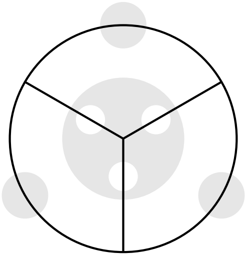

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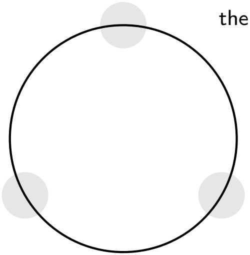
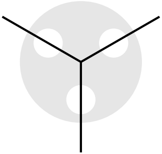
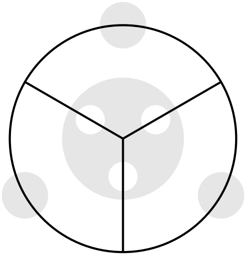






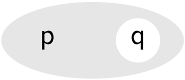
$$\exists x, y, z : x \neq y \wedge y \neq z \wedge z \neq x$$

there are at least three different things

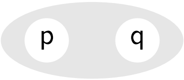
there exist exactly three things

Intermediary Remarks

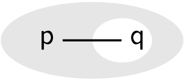
- reoccurring patterns



implication



disjunction



"All p are q"

- juxtaposition is commutative & associative (free ride!)

- canonical representation for FOPC with equality

$$
\begin{aligned}
& p \to (q \vee r \vee s) \\
\equiv\ & (p \wedge \neg q) \to (r \vee s) \\
\equiv\ & (p \wedge \neg q \wedge \neg r) \to s
\end{aligned}
$$

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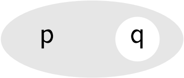

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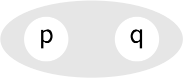

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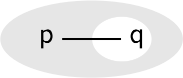

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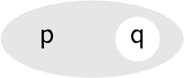

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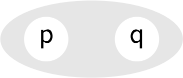

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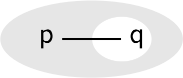
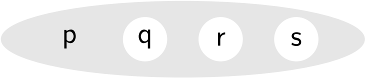


"All p are q"

- juxtaposition is commutative & associative (free ride!)

- canonical representation for FOPC with equality



$$p \to (q \vee r \vee s)$$
$$\equiv (p \wedge \neg q) \to (r \vee s)$$
$$\equiv (p \wedge \neg q \wedge \neg r) \to s$$

Endoporeutic Semantics

- ➪ map to standard FOPC syntax and its semantics and back ? ☹

- ➪ "outside-in" evaluation method deriving truth value of a EG
- ➪ lazy evaluation (can ignore certain subgraphs)

- ➪ equivalent to two-person zero-sum perfect-information game
- ➪ player graphist (proposer) vs. grapheus (sceptic)
- ➪ start with one EG and a Tarski style model $M = (D, R)$
- ➪ EG evaluates to true in M iff graphist has a winning strategy

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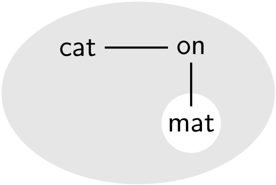
Example

Does



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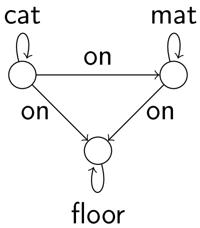

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Endoporeutic Semantics
Game

repeat until one player wins:

- if EG is empty sheet of assertion, then graphist wins
- if EG consists of one negative area,
  then remove outermost cut and reverse roles of players
- if EG consists of more than one negative areas,
  sceptic chooses one and erases all others

  laziness!
- if EG consists of subgraph G without negations and one or more negated graphs, then the graphist proposes a graph homomorphism of G into M (not neccess. isomorphism!)
  - if not possible, then grapheus wins
    else this mapping leads to a partial homomorphism for future moves by assigning the same entity in D to the same line of identity, and we repeat the game on the EG where G is erased

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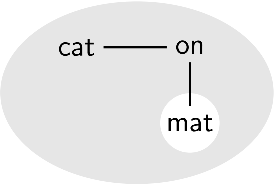
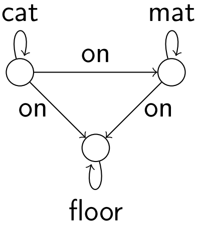
Does 

hold in 

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basic axiom : empty sheet of assertion

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➪ sequence of rule applications:
axiom

recall:

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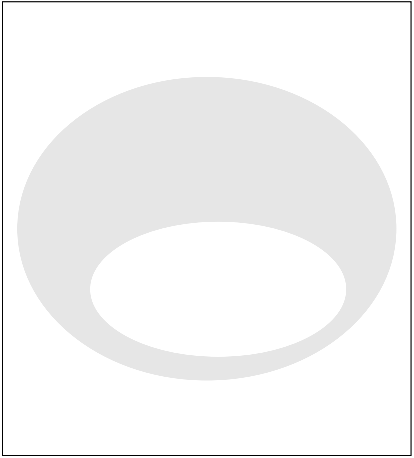

- sequence of rule applications: axiom; 3i

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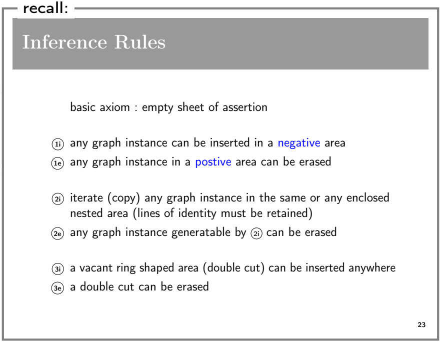

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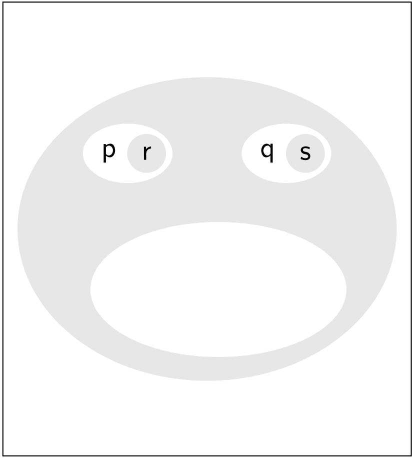

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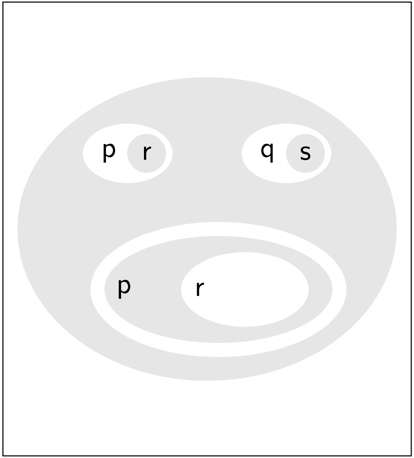

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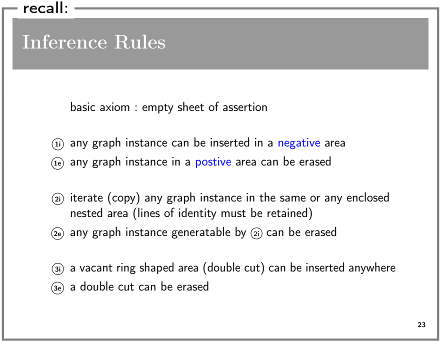

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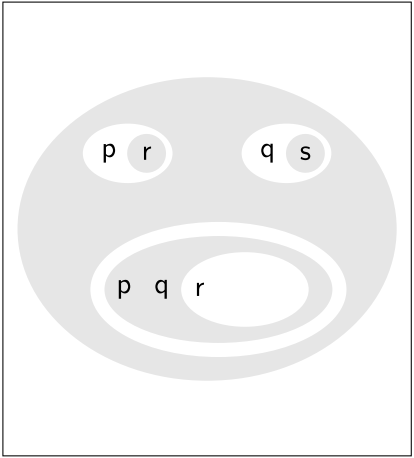

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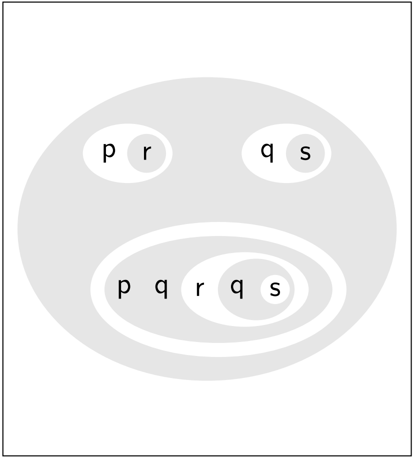

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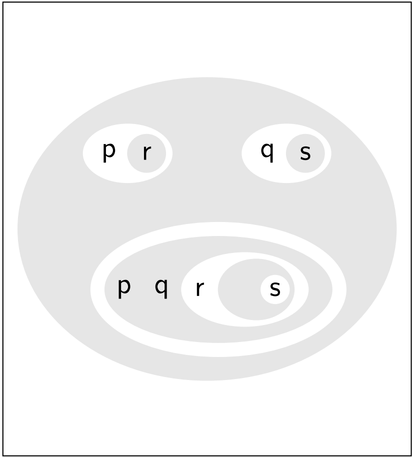

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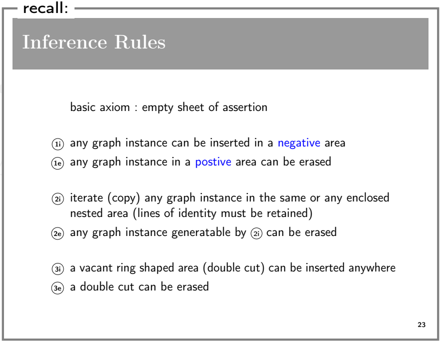

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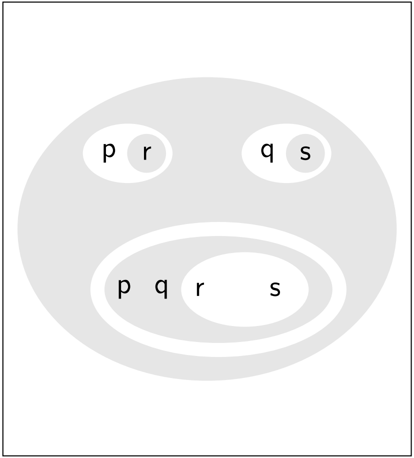

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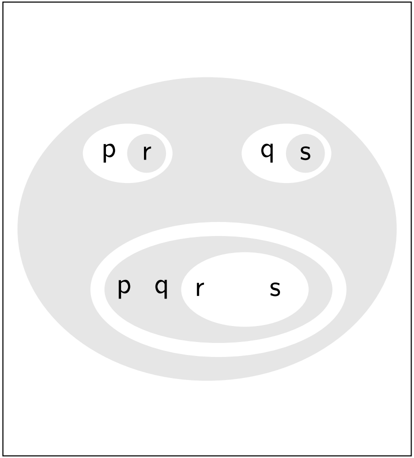

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- derive valid formula: $[(p \to r) \wedge (q \to s)] \to [(p \wedge q) \to (r \wedge s)]$
- Leibniz's Preaclarum Theorem
- ⚠ [Principia Mathematica] needed proof in 43 steps (based on 5 axiom schemata, i.e., inf. number of axioms)

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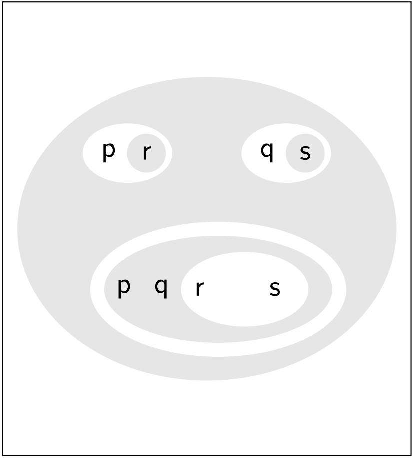

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✎ Can you proof that $a \rightarrow (b \rightarrow a)$ is valid ?

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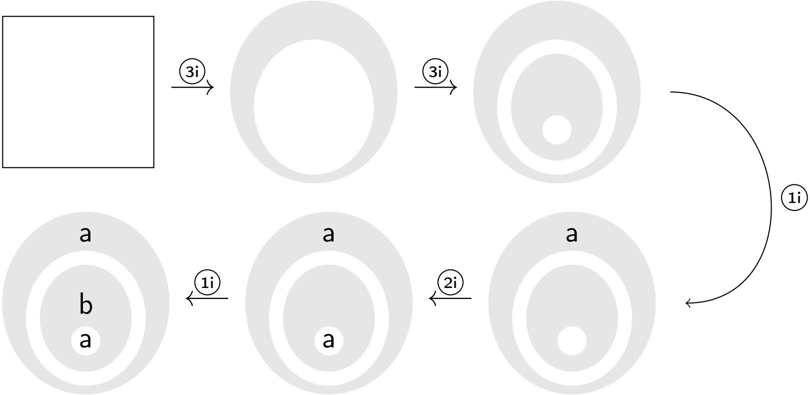

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✎ Can you derive Frege's rule of *modus ponens* ?

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➪ now can apply modus ponens in diagrammatic proofs ☺

Cut-and-Paste Theorem

remark:

- inference rules only depend on whether area is negative/positive
- independent of how deeply nested

Cut-&-Paste Theorem

any proof possible on empty sheet of assertion can be “cut out” and “pasted” in any positive aread of any depth

Re-inventing the Logical Wheel

Deduction Theorem

if q can be derived from p then $p \to q$.

Re-inventing the Logical Wheel

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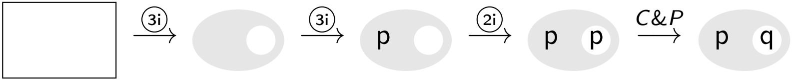

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We saw...

- system of EG "equivalent" to FOPC with equality
- purely diagrammatic rules of inference ("draw a proof")
- most simple & symmetric formulation of FOPC's inference rules
- "intuitive" formalization language (really ?)

But there is more !

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- graph-transformation view of inference and proofs...
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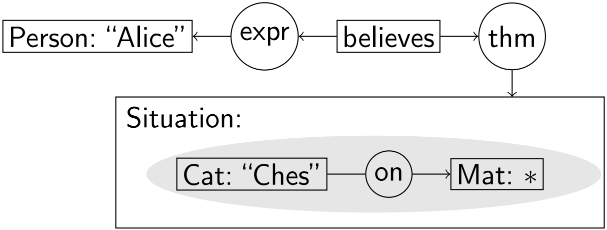
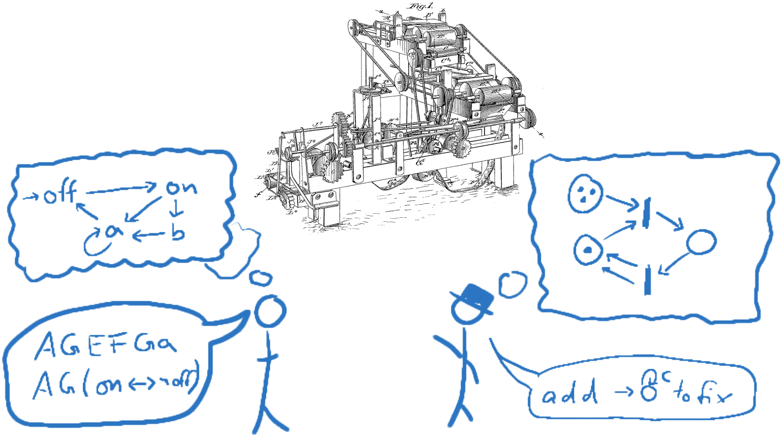
Derived Notions

- Peirce's γ-graphs (modal logic)
  ➪ introduce new kind of cut: p for "$\Box p$"
  ➪ extend inference rules. . .

- Sowa's conceptual graphs (knowledge representation)
  ➪ add types, type hierarchy, unique individuals,. . .
  ➪ distinguish concepts (boxes) & relations (circles)



Back to Formal Methods



Open Questions

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- formalization depends on (semi-)formal language
- μ-calculus et al. are expressive, ideal for algorithmic treatment, but not intuitive (= "close" to mental model)

- operational/relational model leads directly to graph-based models and graph-based reasoning
- why not focus directly logics on graphs/graph constraint systems/graph reasoning/etc. ?

- there are powerfull (wrt. to expressiveness, reasoning, etc.) formal diagrammatical languages !
- what are appropriate diagrammatical logics in our domain ?

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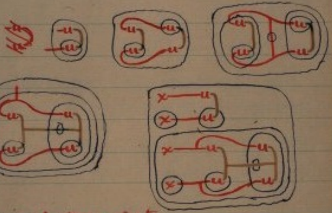
~~The whole theory of numbers~~
depends upon five premisses represented in this graph



The red ligatures refer to numbers, the brown ligatures to any universe such that there is a relation that u may be understood to express, that will make the entire graph true.

x is to be understood to be replaceable by any monad graph whatever without altering the truth of the ~~en~~ entire graph.

from Peirce's MS 464

References / Thanks

- Peirce's MS 514 with extensive commentary by J. Sowa
  `http://www.jfsowa.com/peirce/ms514.htm`

- comprehensive bibliography for EG
  `http://existential-graphs.org/eg_readings.shtml`

- MS464 from `http://www.unav.es/gep/Port/ms464/9.html`
- pictures (logicians, machine,...) from public domain