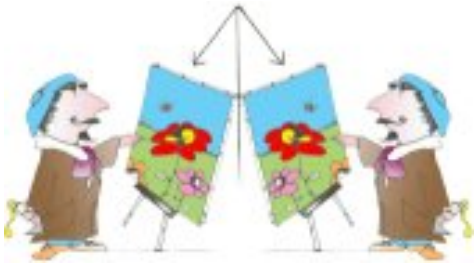

ULB

Model Checking Communicating Processes: Run Graphs, Graph Grammars & MSO

Alexander Heußner

Formal Methods & Verification group Brussels

GT-VMT 2012

Agenda

- formal model of distributed recursive communicating processes
- recall un-/decidability of basic verification questions
- show line of attack via under-approximative approaches

- from semantic "boundedness" to graph grammars
- and back to decidability of MSO model checking

article takes different way:

- try to extend known results beyond reachability
- directly arrive at undecidability
- extend proof-idea of reachability result for subclass of RCPS
- arrive at graph grammars for run graphs
- apply known HRG-treewidth-MSO connection

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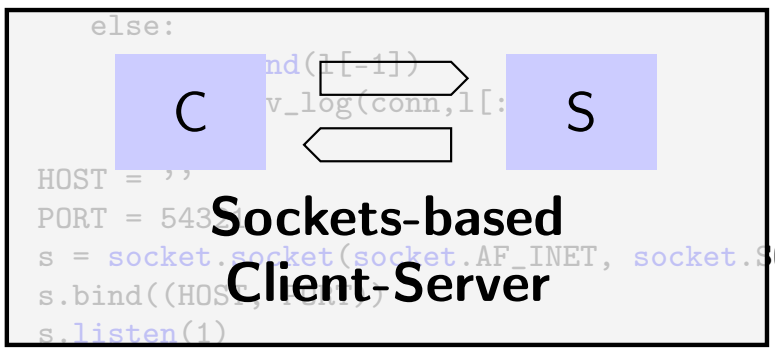


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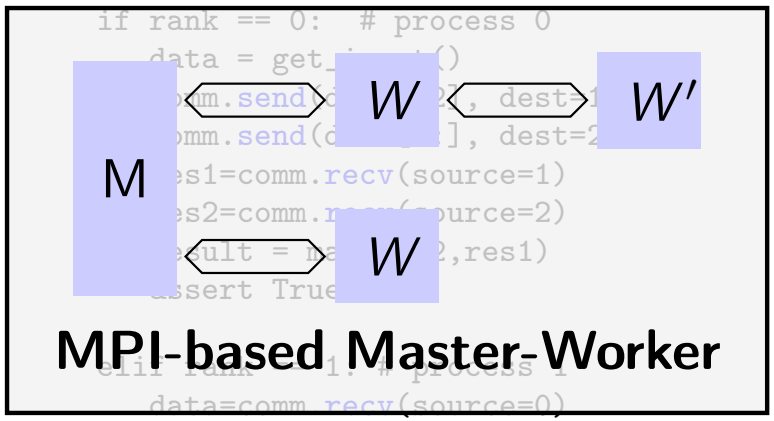

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check safety

Verification of Distributed Systems

check specification in formal logic

e.g., the server returns all received messages in reverse order

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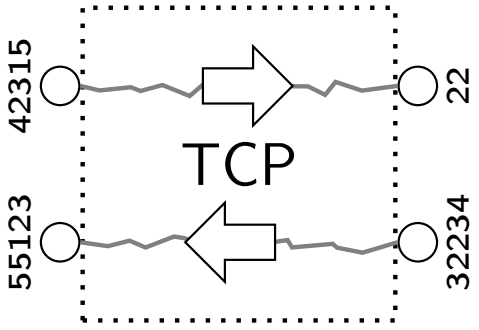
Formal Model : RCPS



Formalize by Recursive Communicating ProcesseS

- reliable, asynchronous, unbounded channels $\longmapsto$ fifo queues
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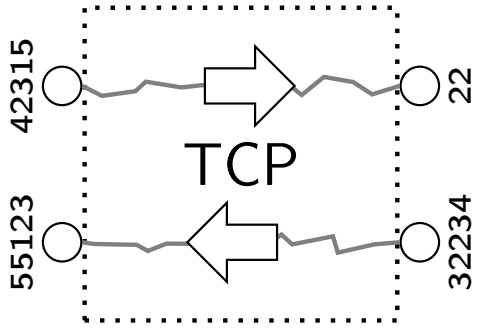

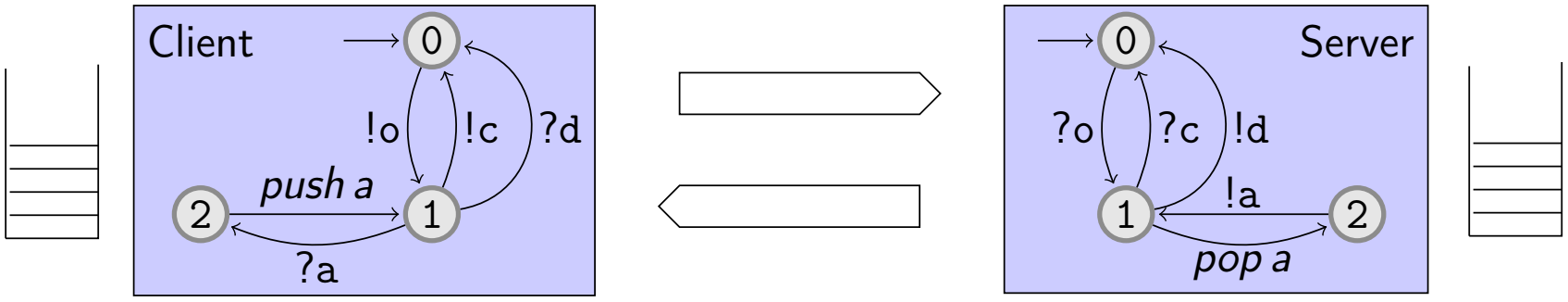

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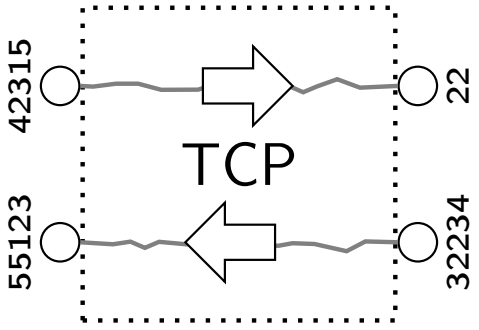

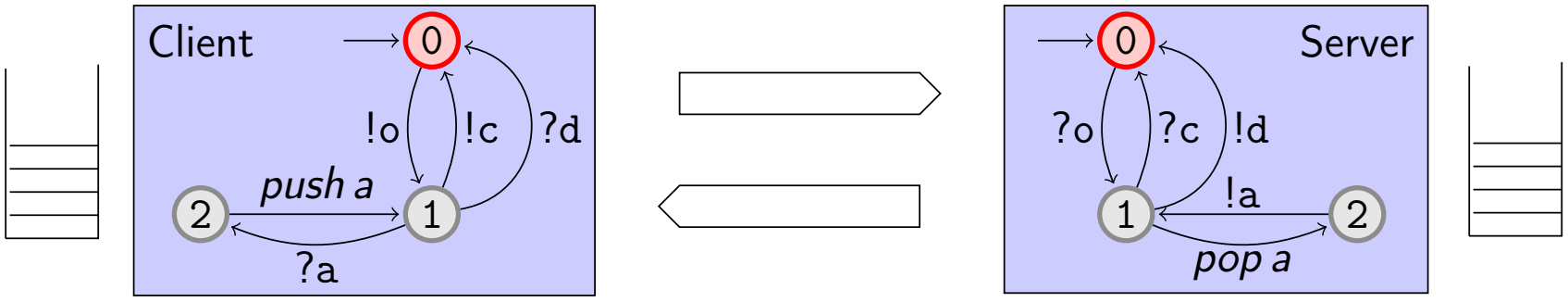

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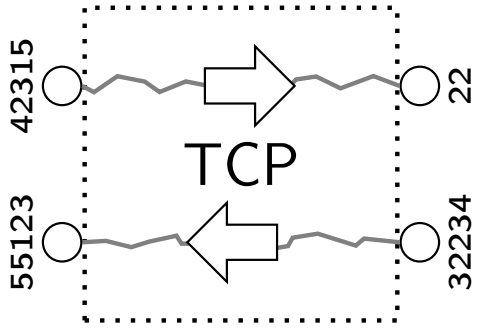

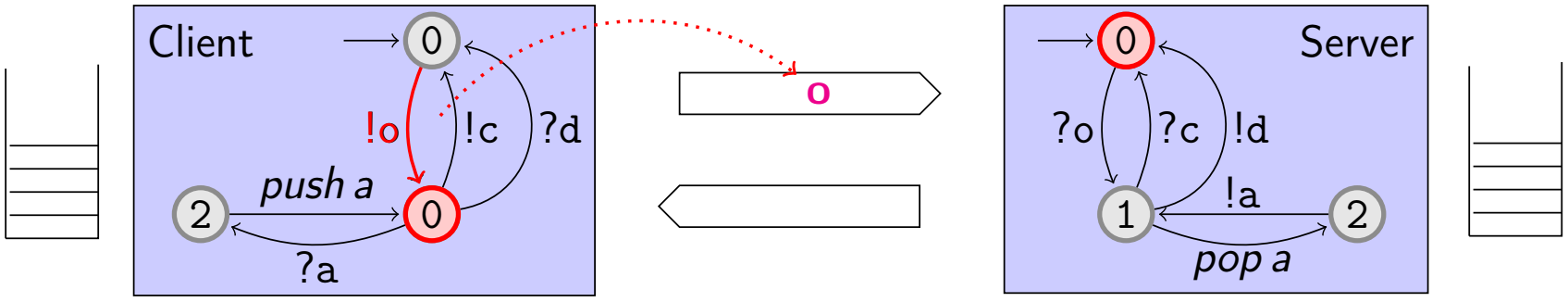

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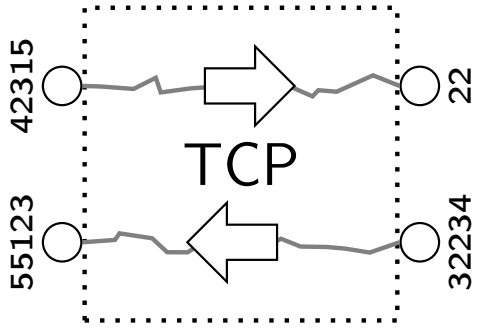

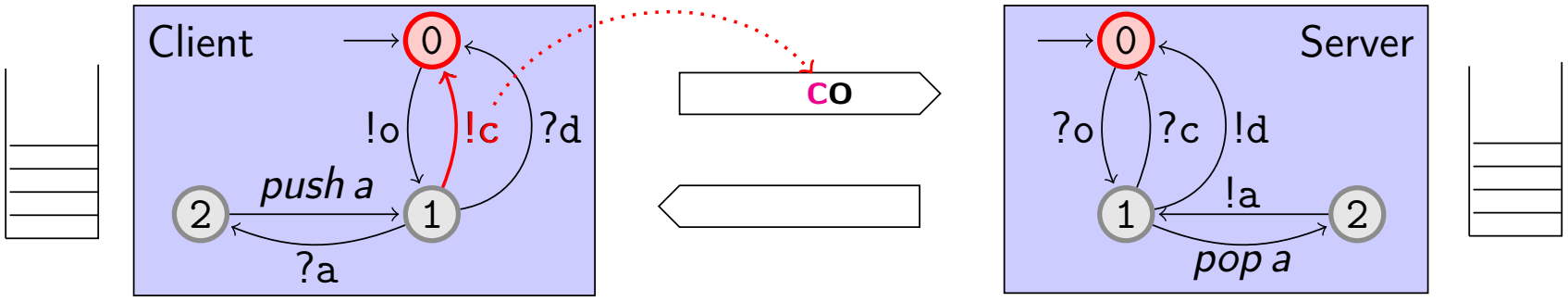

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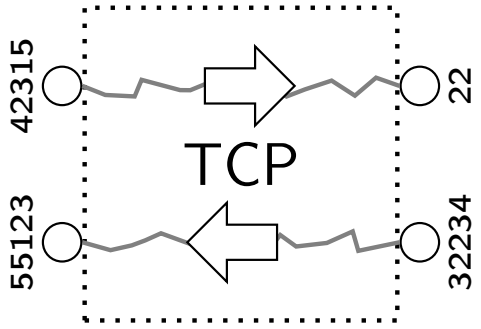

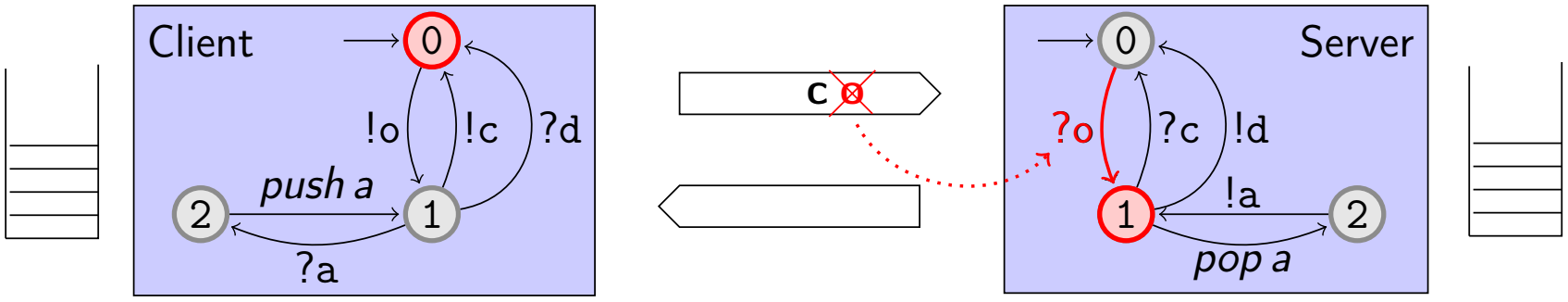

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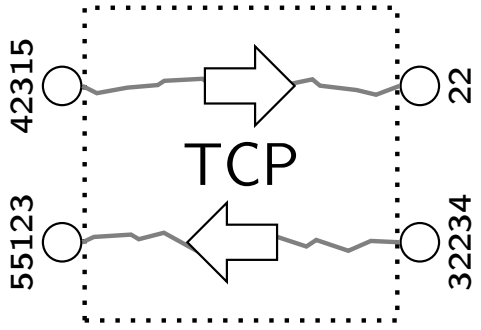

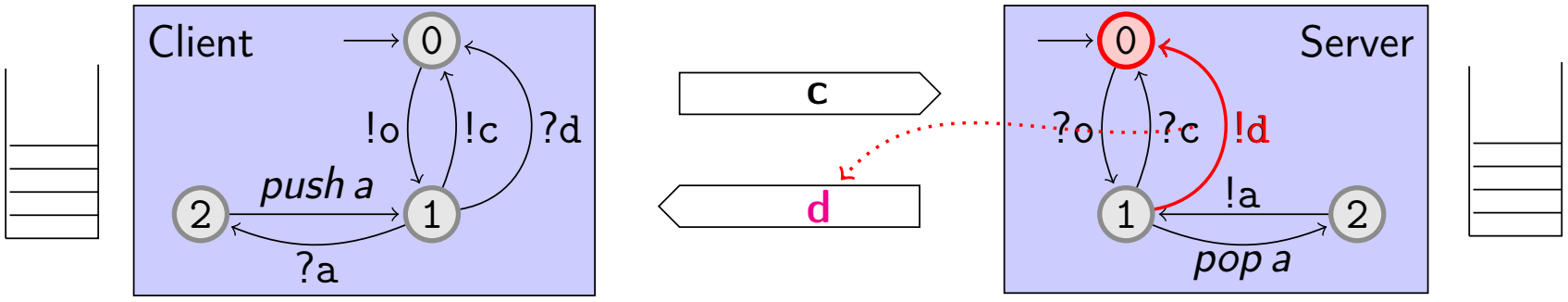

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Reachability Verification

- reachability undecidable for finite-state RCPS ↯

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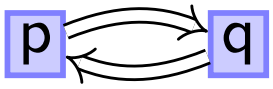
  & 

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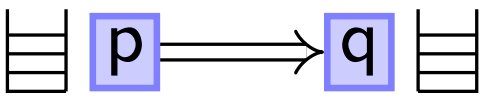
  already for 

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Idea:

Restrict focus only on subset of possible runs that have a certain form (under-approximate behaviour).

- ➪ try to find restriction on runs that have
  - (i) good theoretical properties (decidable reachability etc.)
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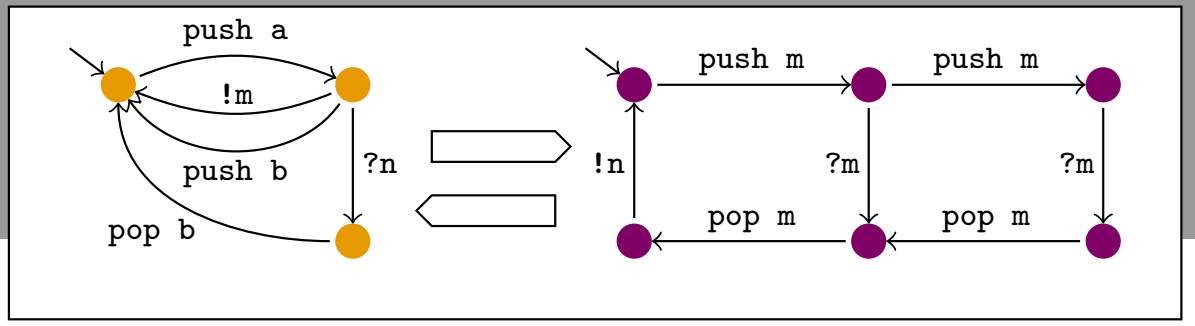

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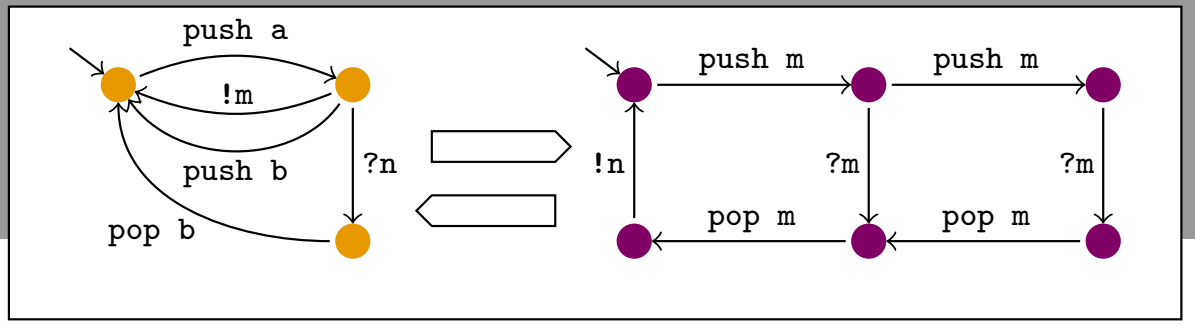

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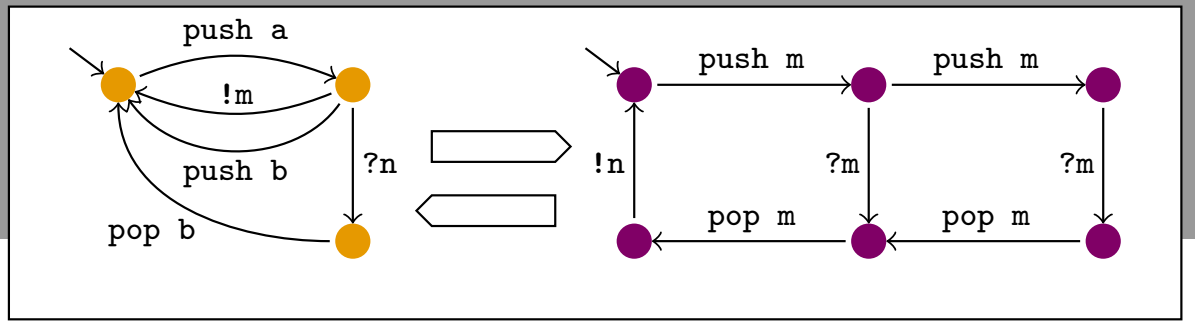

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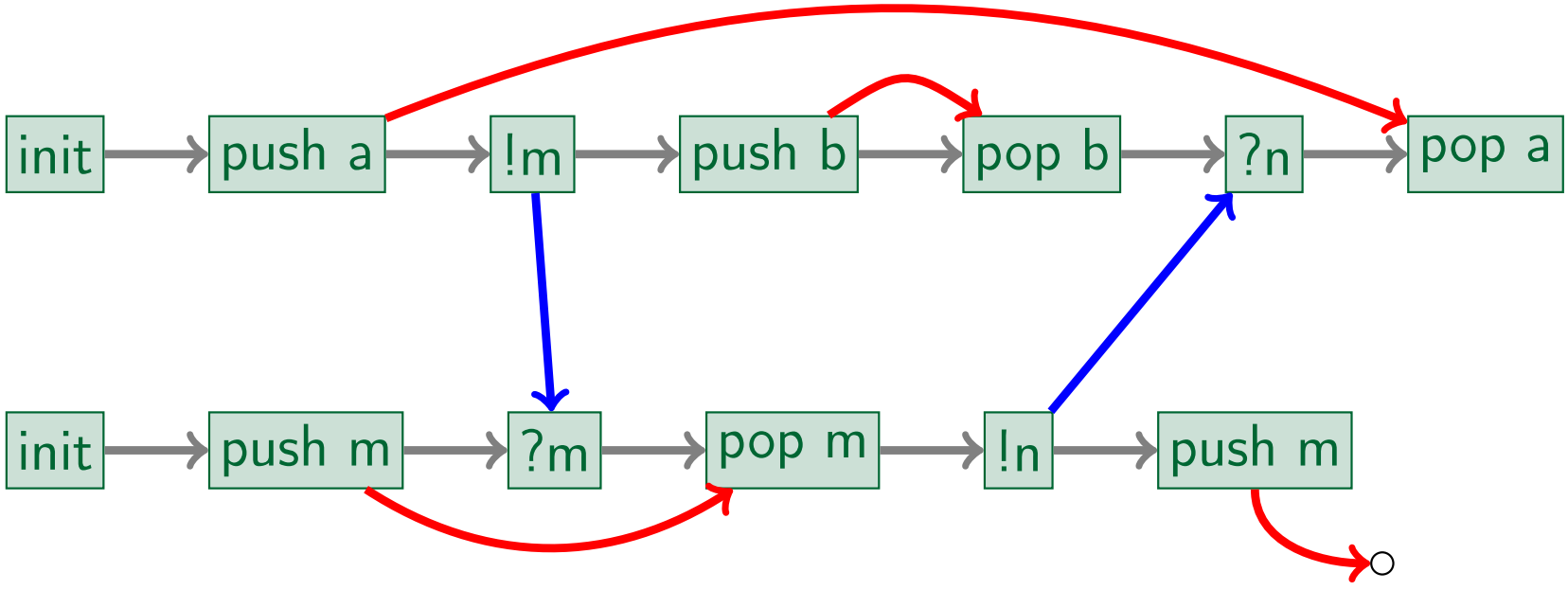

Boundedness Conditions: Interleaving

- “ressource”-boundedness
  - atomic communication
  - untangle pushdown and communication actions
  - architecture: tree-like, acyclic,. . .
  - mixed: half-duplex,. . .

- family of phase-boundedness restrictions:
  e.g., contexts [Qadeer/Rehof’05]

- reachability decidable in $\mathcal{O}(\text{ExpTime} / \text{ExpSpace})$
- rarely decidability results beyond reachability

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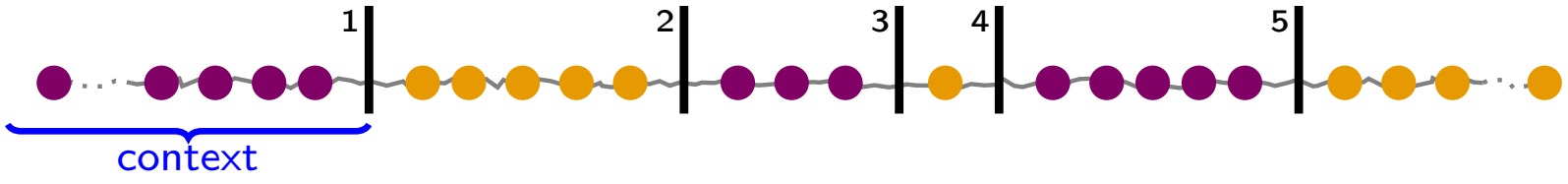

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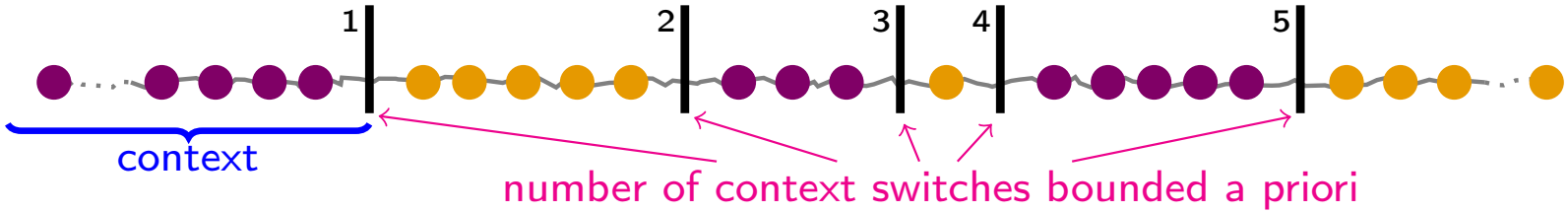

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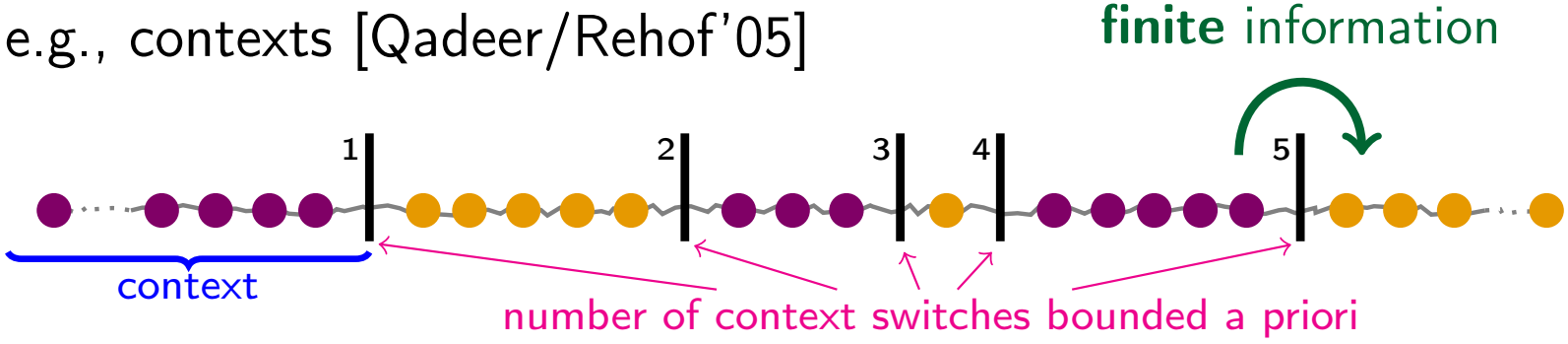

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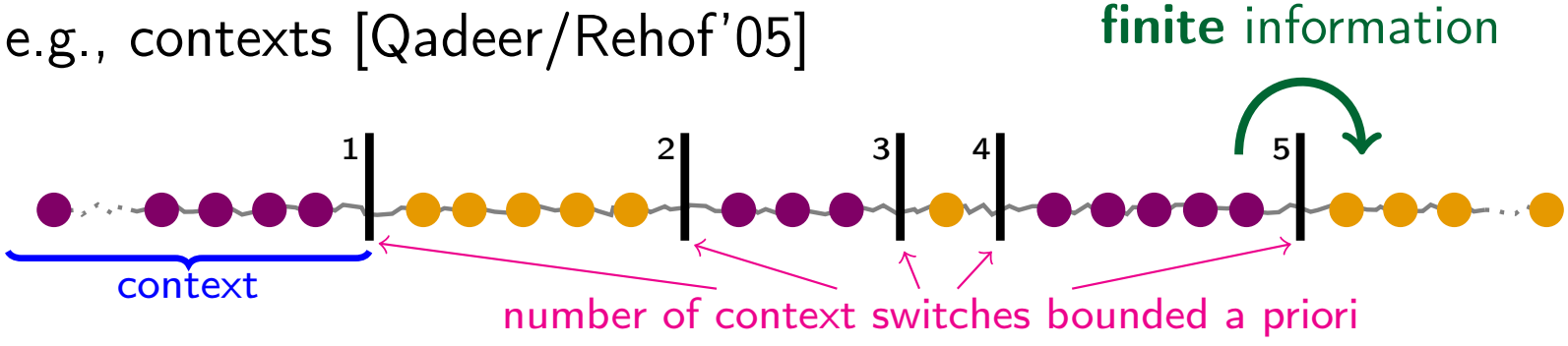

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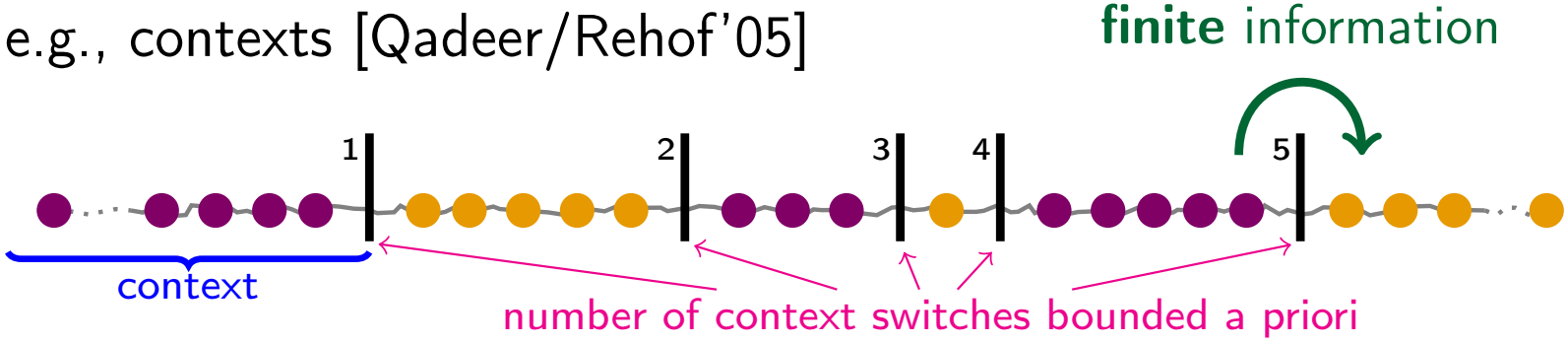

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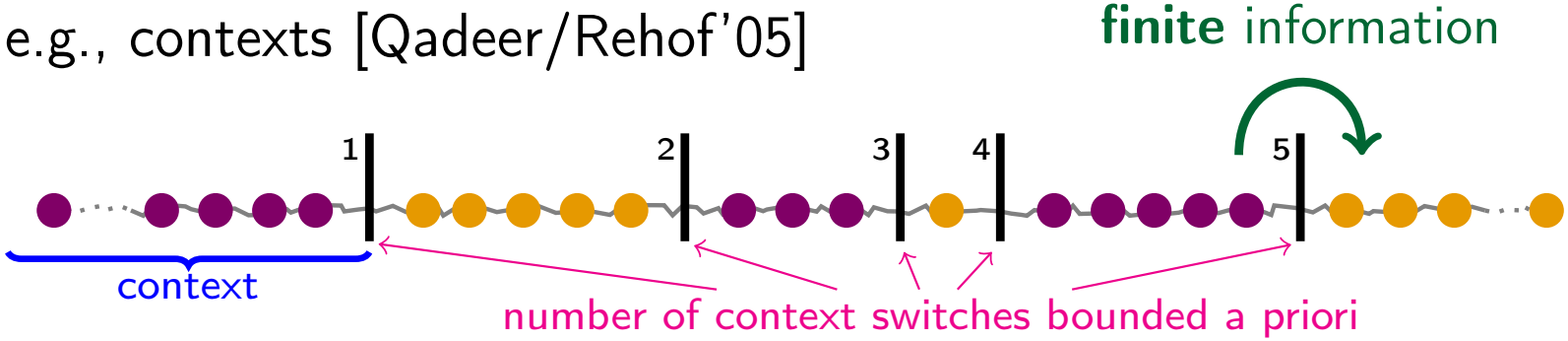

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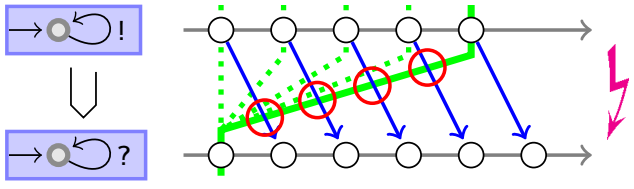

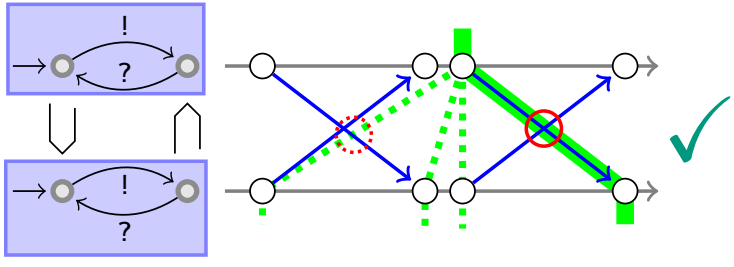

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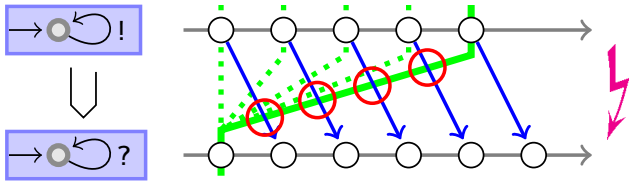

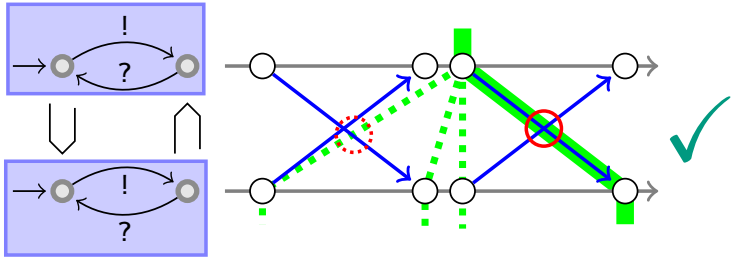

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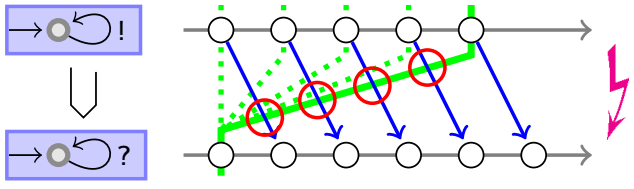
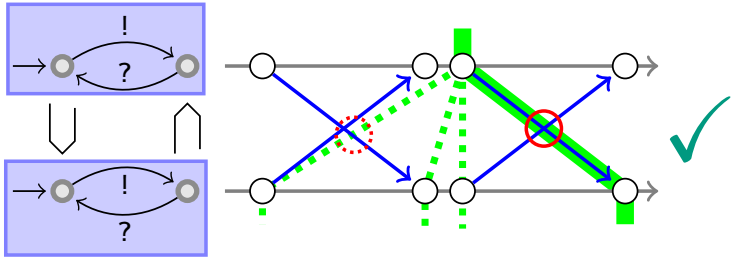
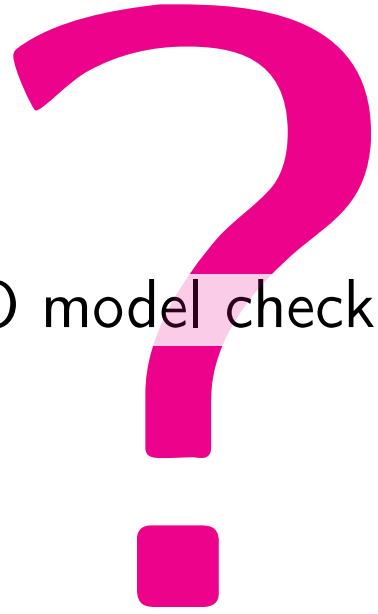
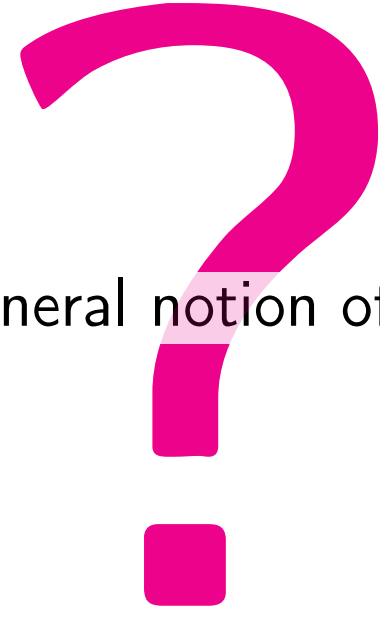




How to extend MSO model checking ideas to RCPS ?



Derive more general notion of boundedness ?



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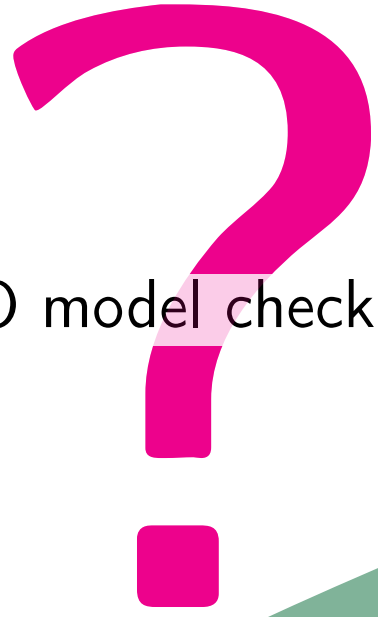

goal: proof one, get one for free !

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Again: Undecidability for finite RCPS

Theorem [Seese '75] :

Let $\mathcal{C}_G$ be a class of graphs such that for every integer $k > 1$ there is a graph $G \in \mathcal{C}_G$ such that G has the $k \times k$ grid as induced subgraph[1]. Then MSO is undecidable on $\mathcal{C}_G$.

- MSO undecidable on rungraphs of finite-state RCPS as one can embedd the $k \times k$ grid already on

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$$G_{3\times 3} = \begin{array}{ccc} 1 & \to 2 & \to 3 \\ \downarrow & \downarrow & \downarrow \\ 4 & \to 5 & \to 6 \\ \downarrow & \downarrow & \downarrow \\ 7 & \to 8 & \to 9 \end{array}$$

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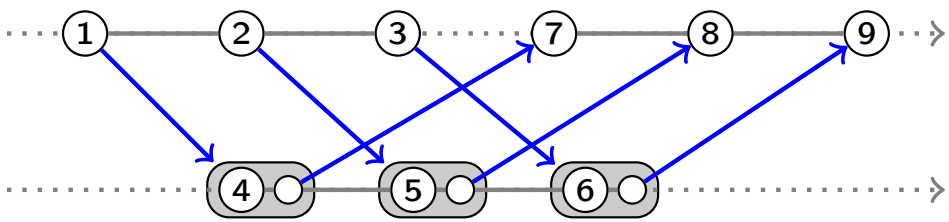

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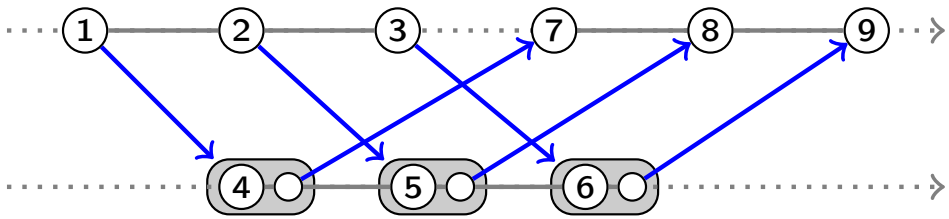

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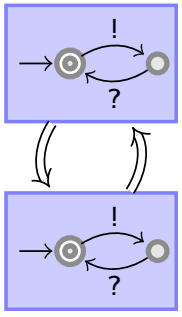

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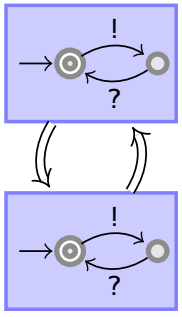

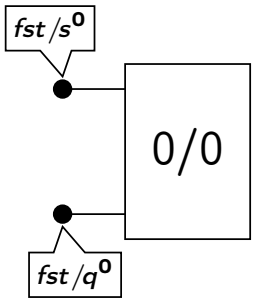

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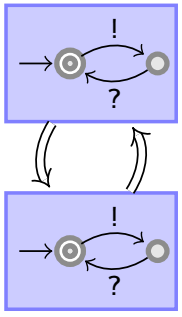

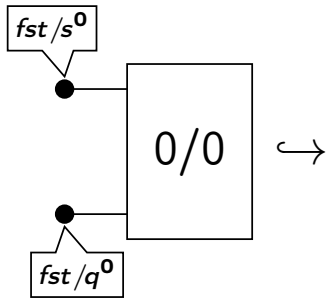

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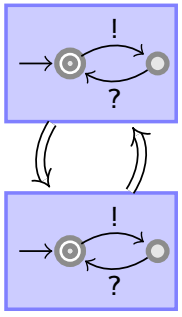

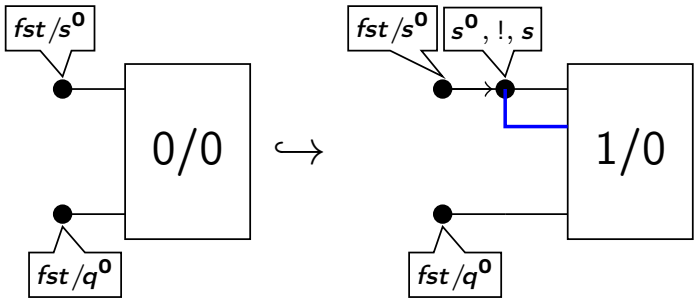

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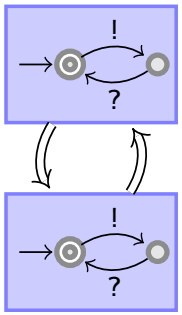

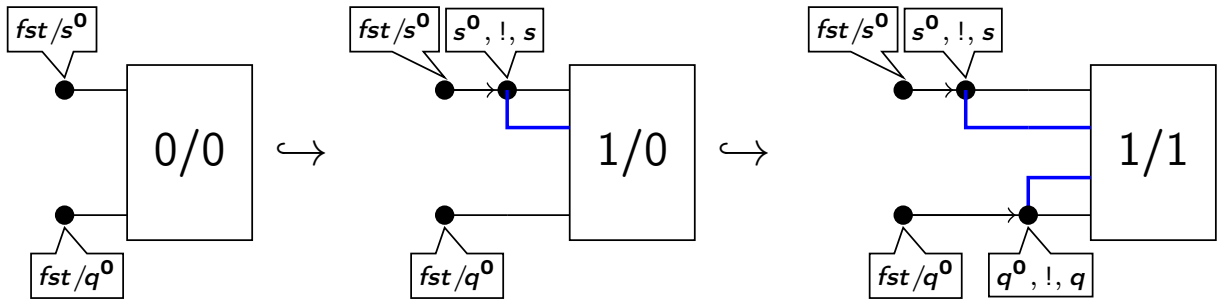

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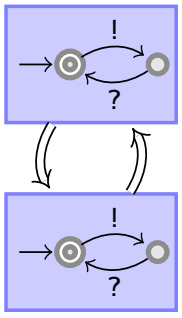

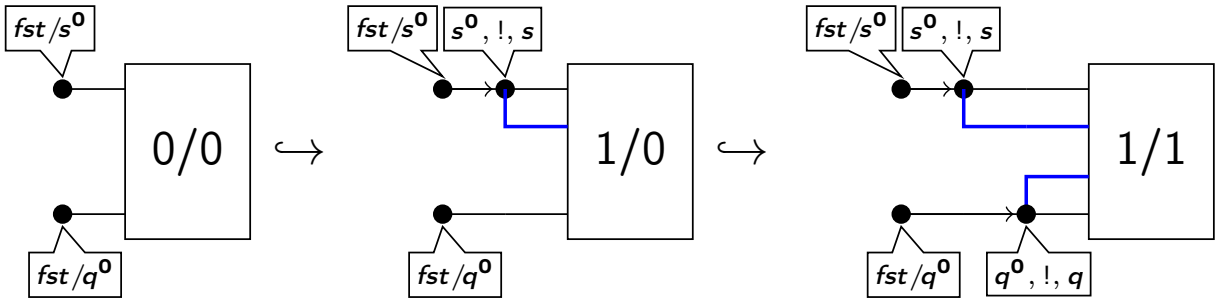

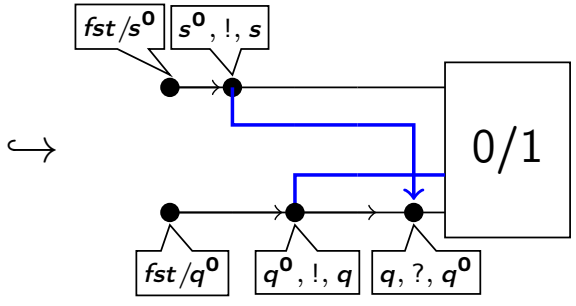

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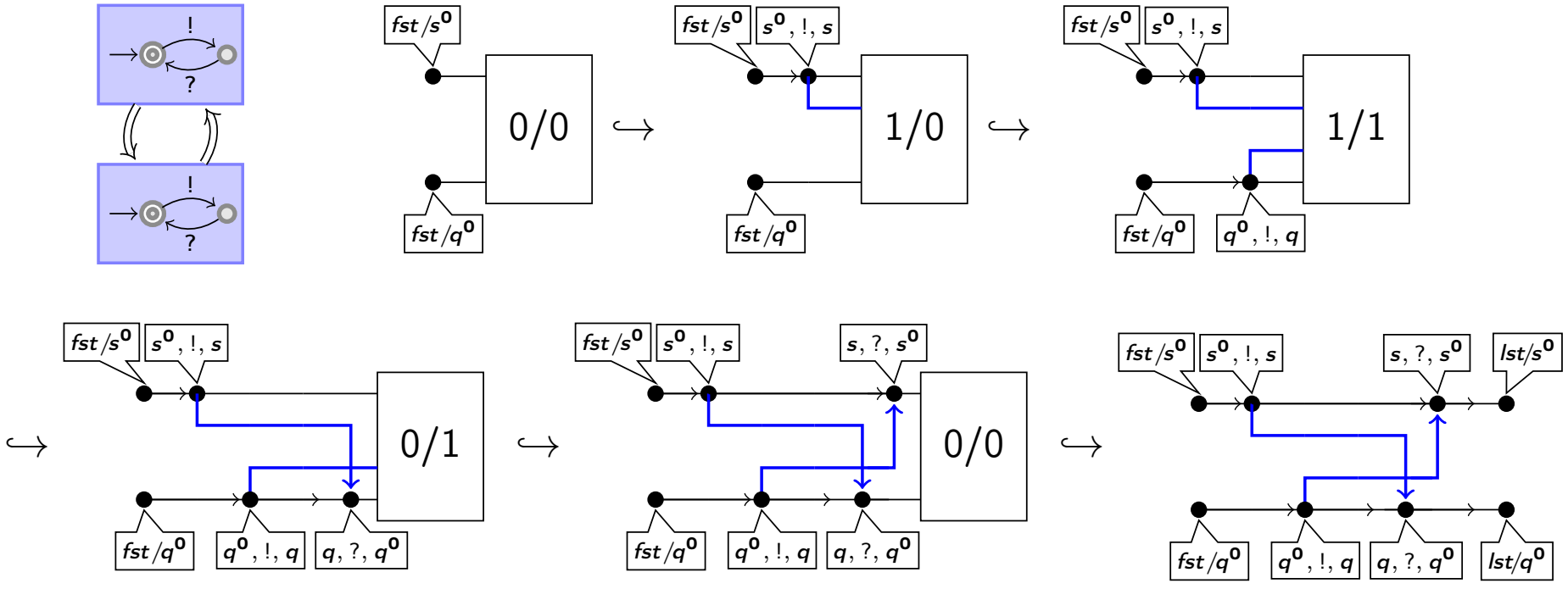
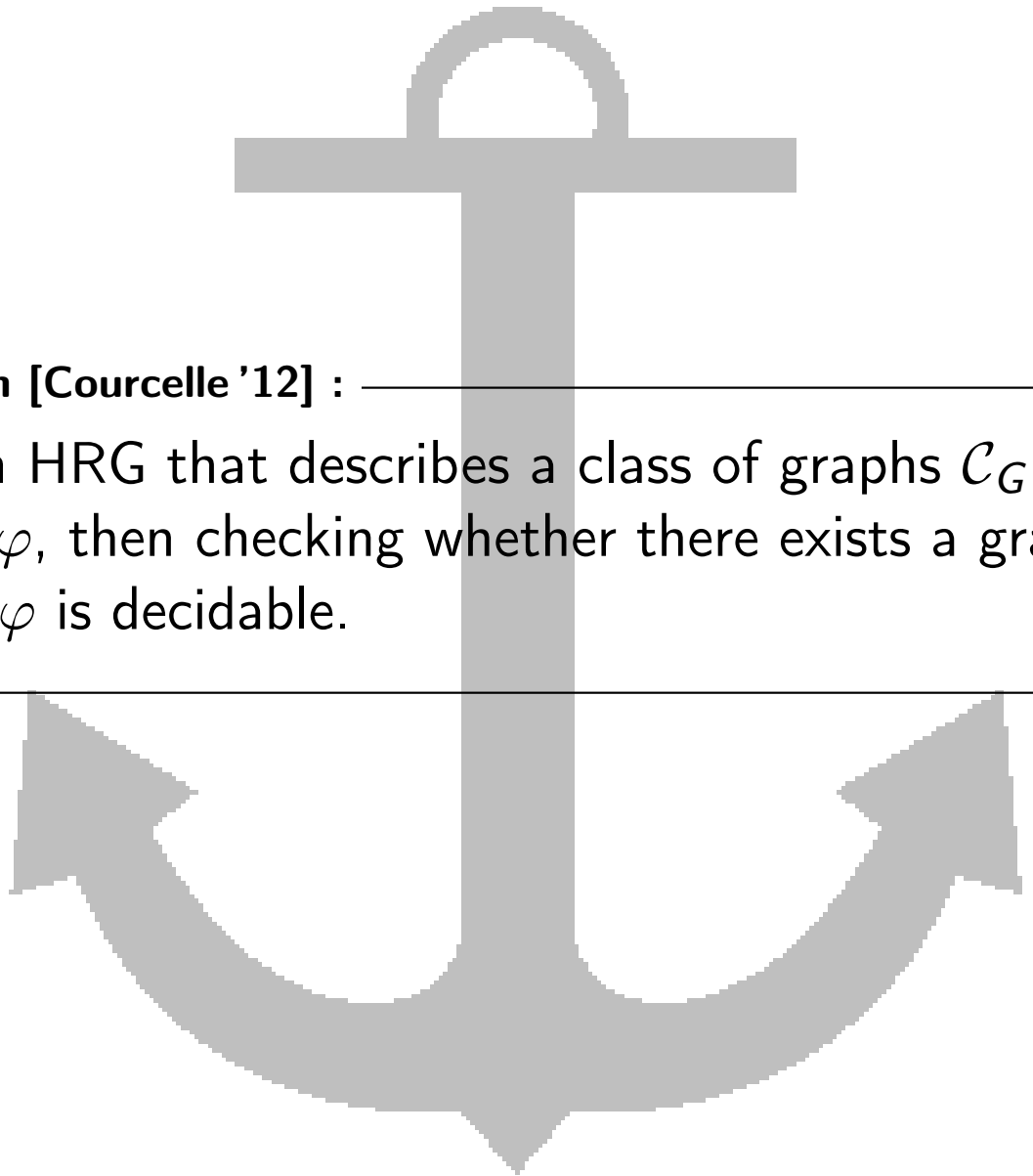


➪ reminds of derivations of (context-free) graph grammars ☺

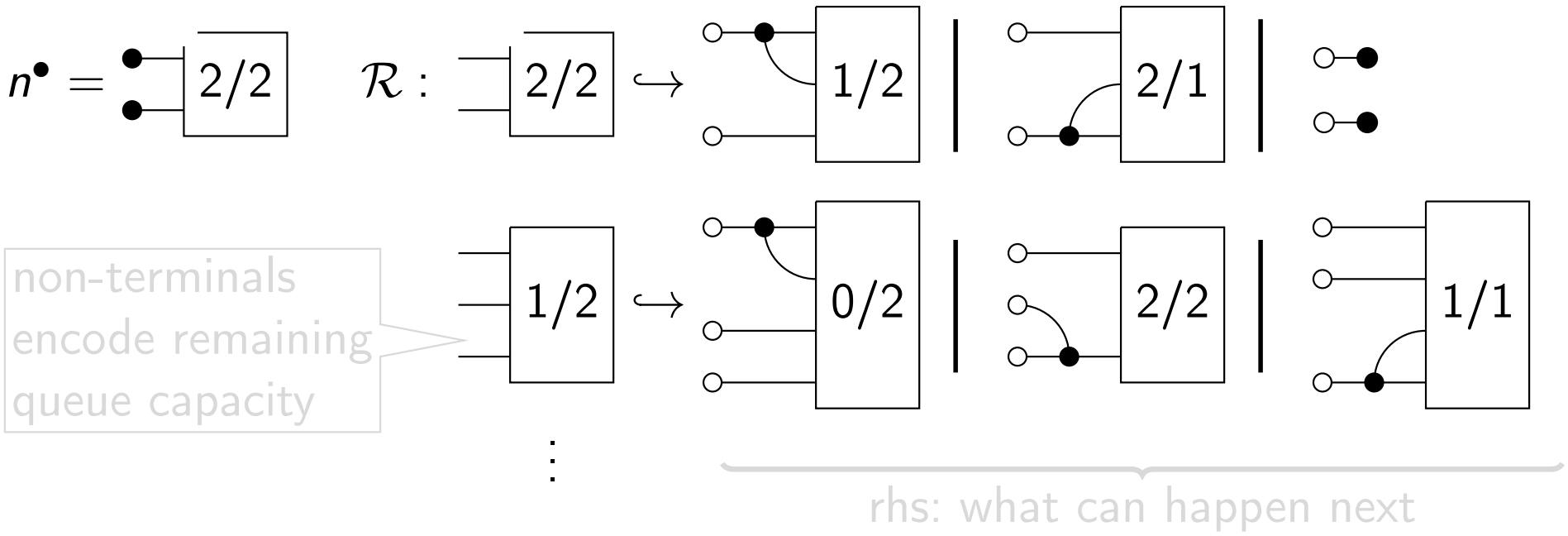
From HRG to MSO Decidability



Theorem [Courcelle '12] :

Given an HRG that describes a class of graphs $\mathcal{C}_G$, and an MSO formula φ, then checking whether there exists a graph in $\mathcal{C}_G$ that satisfies φ is decidable.

A HRG for bounded finite RCPS



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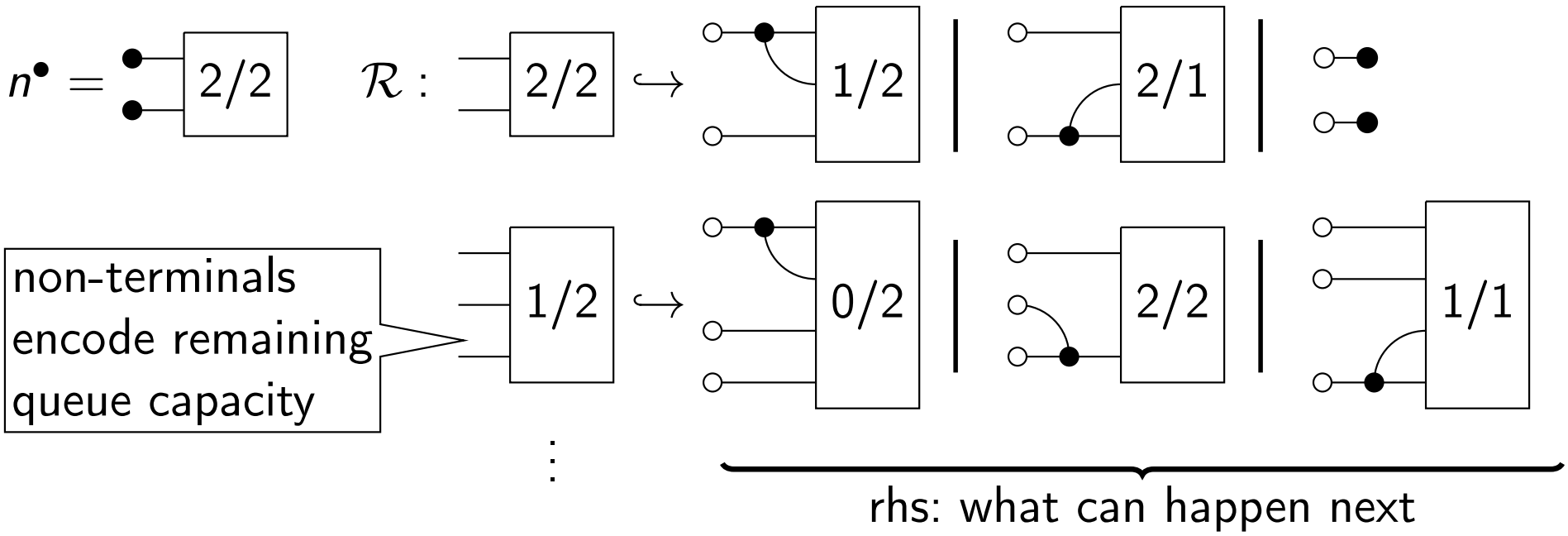

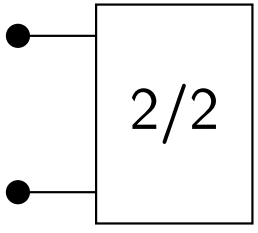

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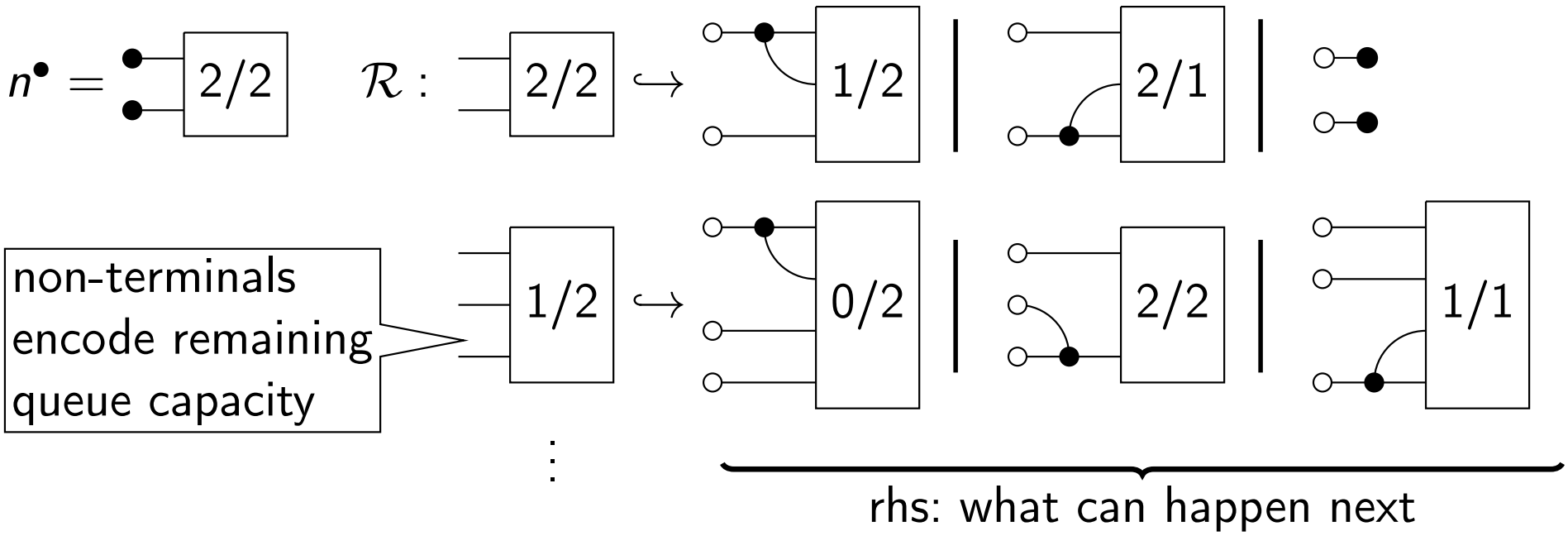

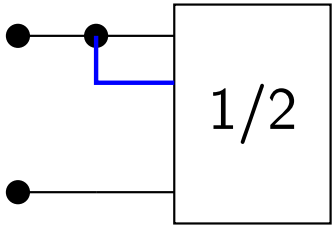

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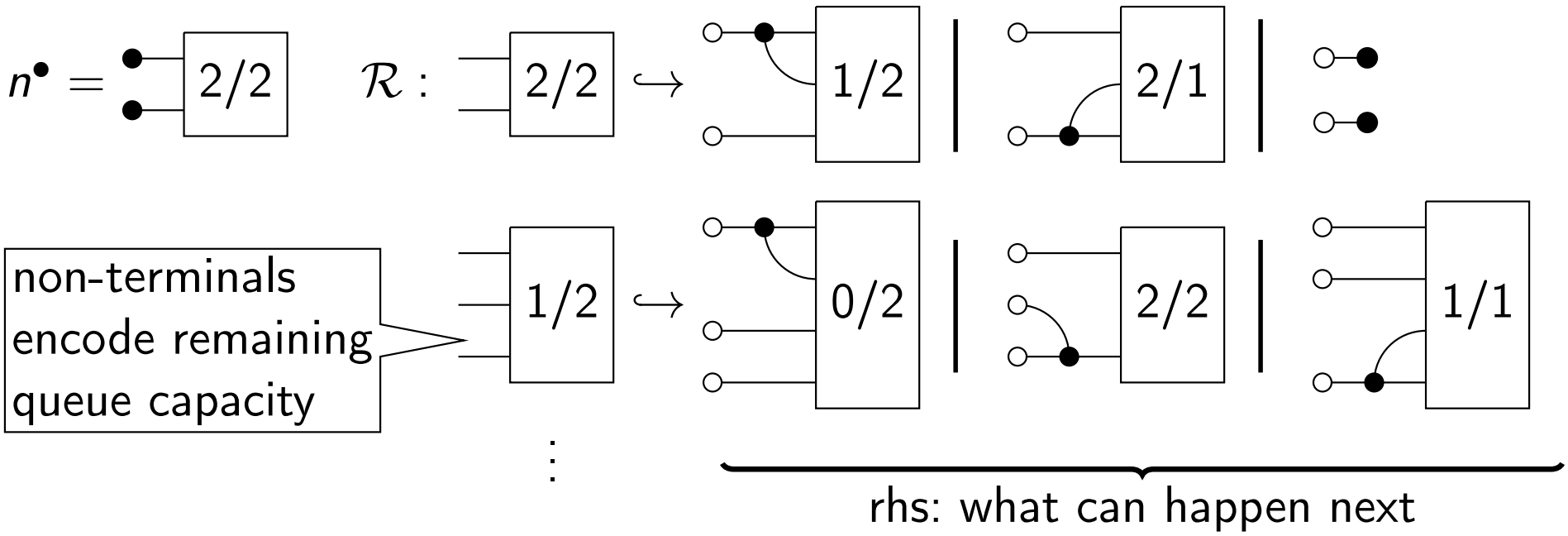

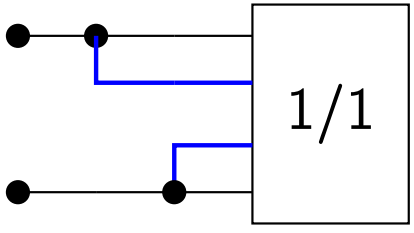

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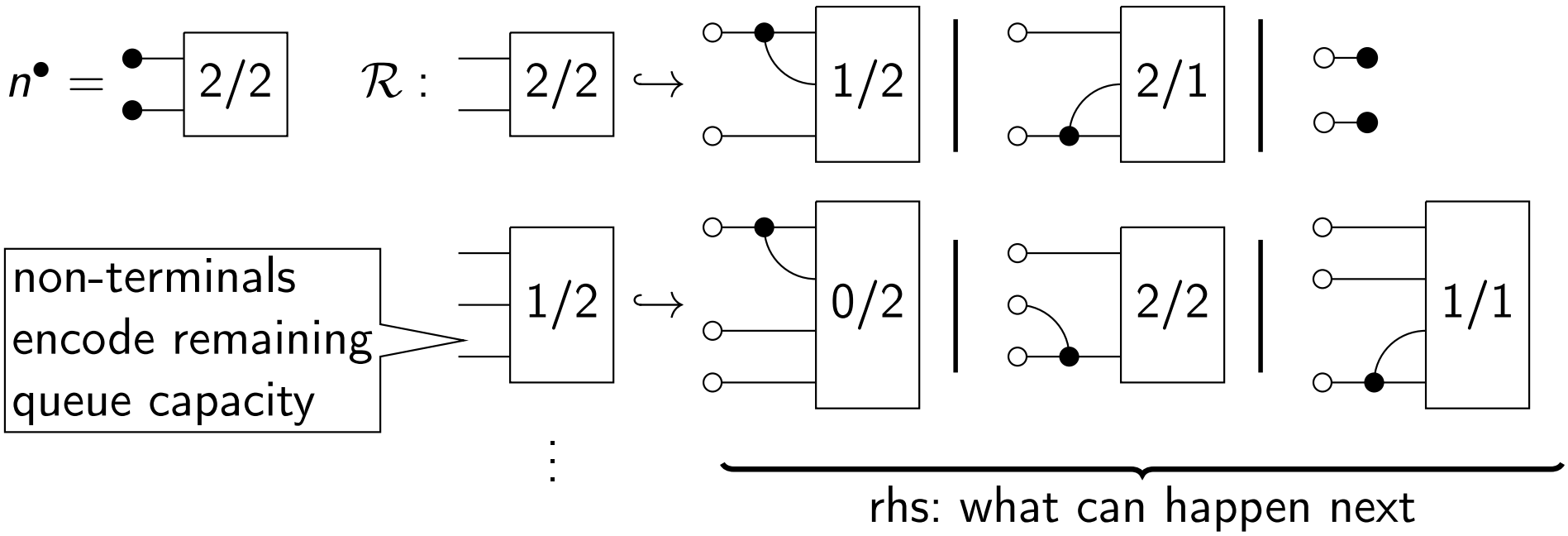

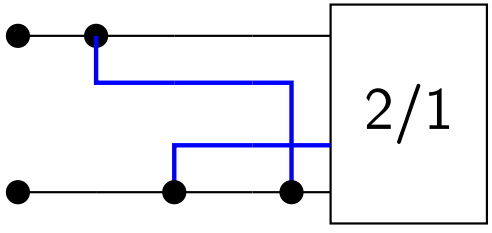

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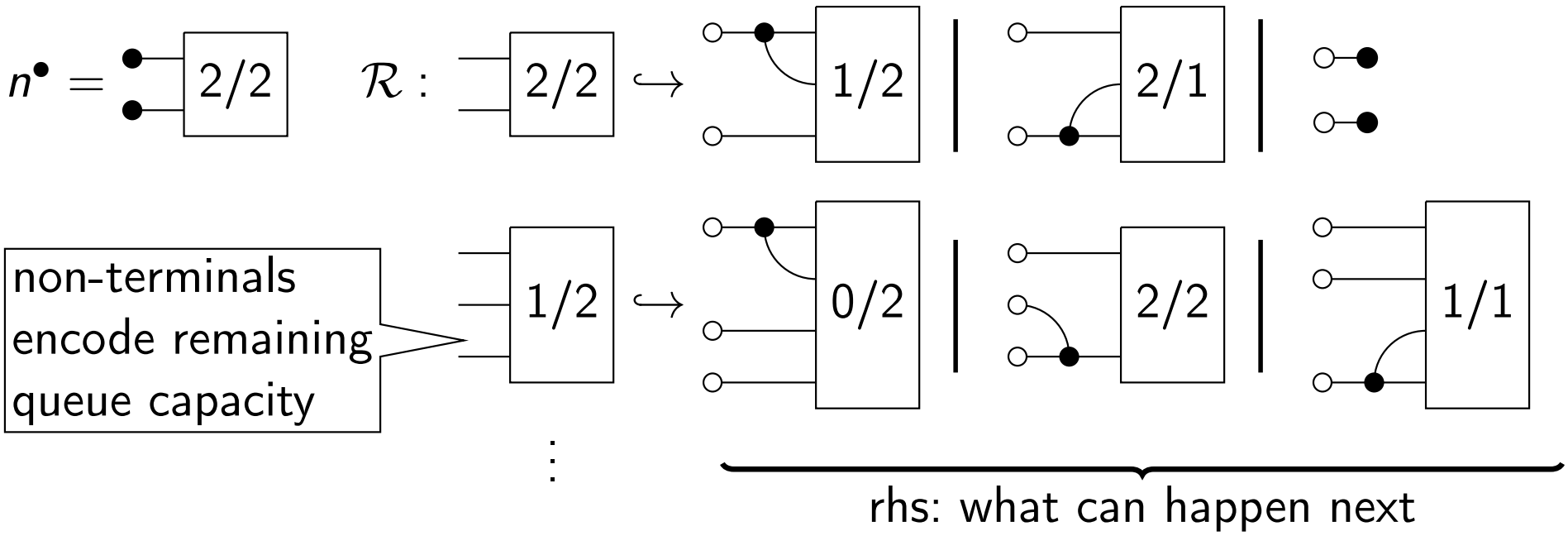

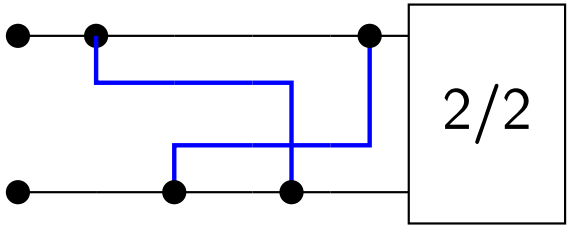

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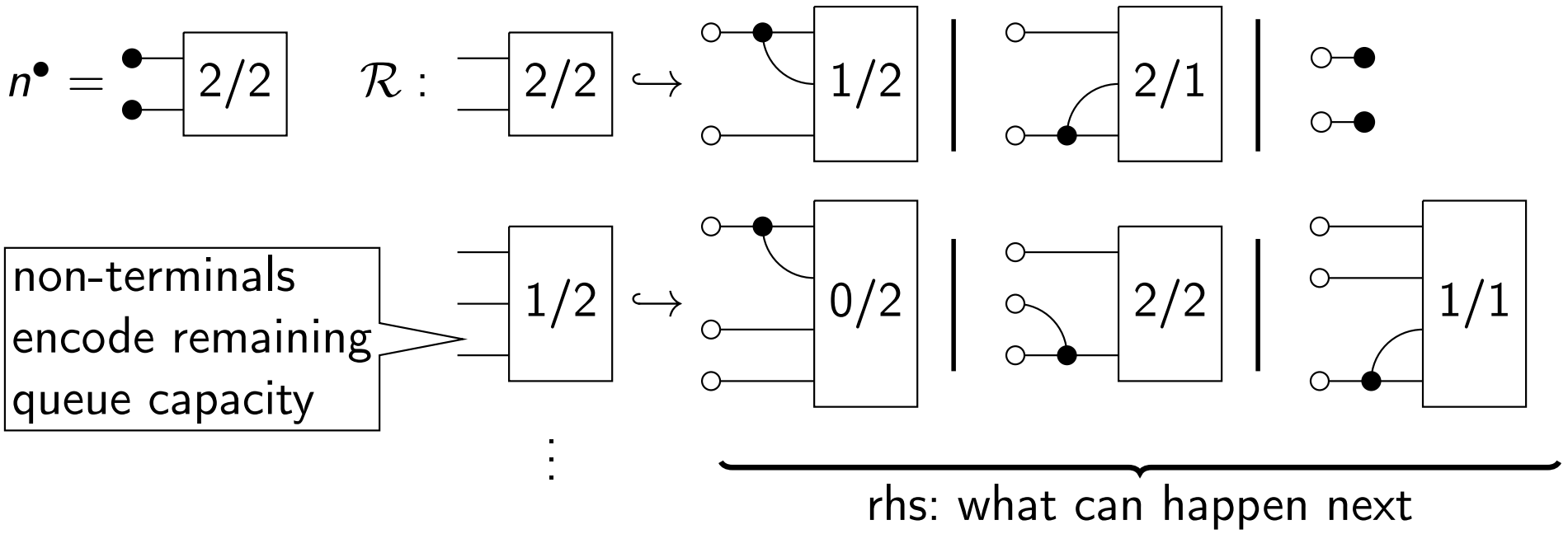

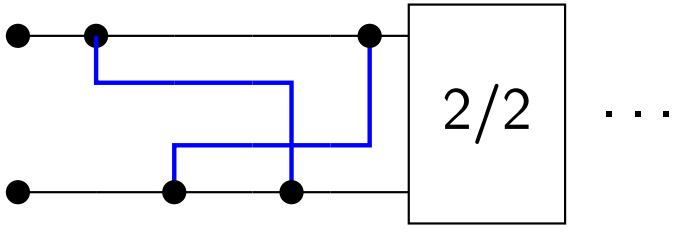

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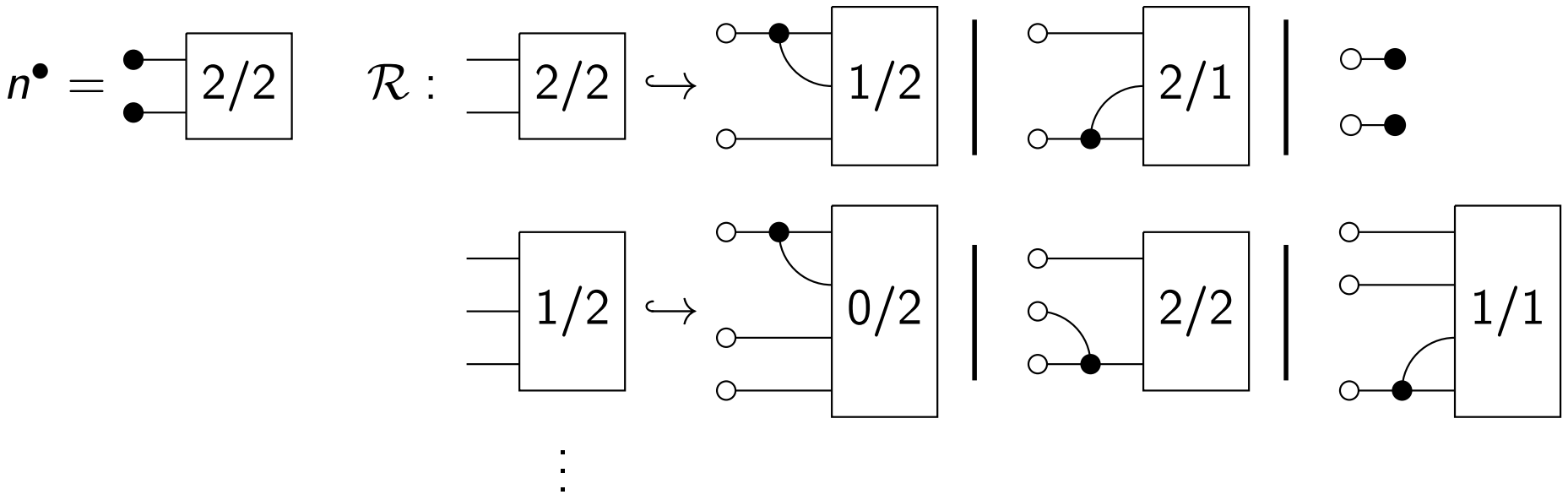

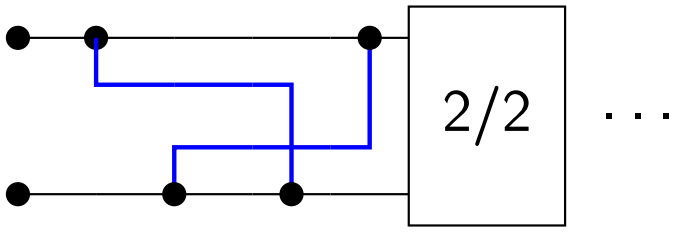

- HRG generates rungraphs of 2-bounded finite RCPS of 2 processes
- generalize: p processes and bound k
- rulewidth of run graphs of bounded finite RCPS is in $\mathcal{O}(p \cdot k)$

From HRG to MSO Decidability

Proposition:

The MSO model checking problem is decidable for rungraphs of bounded finite-state RCPS.

➪ already known [Madhusudan'03] [Genest et al.'07],
but here new "style" of proof ☺

➪ by connection of HRG rulewidth with treewidth:

Corollary:

MSC of bounded finite-state RCPS have bounded treewidth.

➪ bounded treewidth more general notion of semantic boundedness ?

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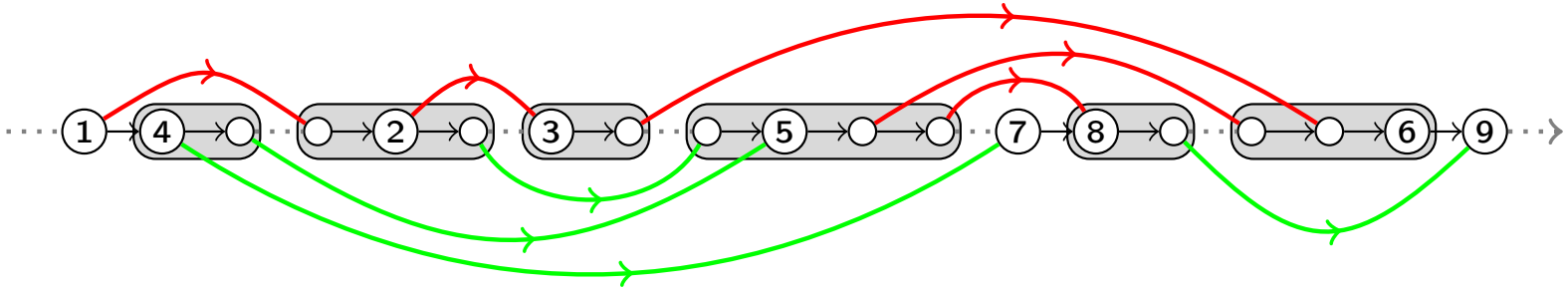
Undecidability for General RCPS

- reachability (and MSO) undecidable for RCPS ↯

  idea: infinite-grid-as-minor argument !

  - look at 2PDS (can reduce RCPS to this case)

$$G_{3\times 3} = \begin{array}{ccccc} 1 & \rightarrow & 2 & \rightarrow & 3 \\ \downarrow & & \downarrow & & \downarrow \\ 4 & \rightarrow & 5 & \rightarrow & 6 \\ \downarrow & & \downarrow & & \downarrow \\ 7 & \rightarrow & 8 & \rightarrow & 9 \end{array}$$



Ressource Bound $\mapsto$ Grammar Bound

Theorem [Heußner et al. 2010]:

Reachability is decidable for RCPS iff they are well-queueing, eager, and imbalanced. (even ExpTime-complete!)

- original proof:
  commutative reordering of interleaving allows 1-stack simulation
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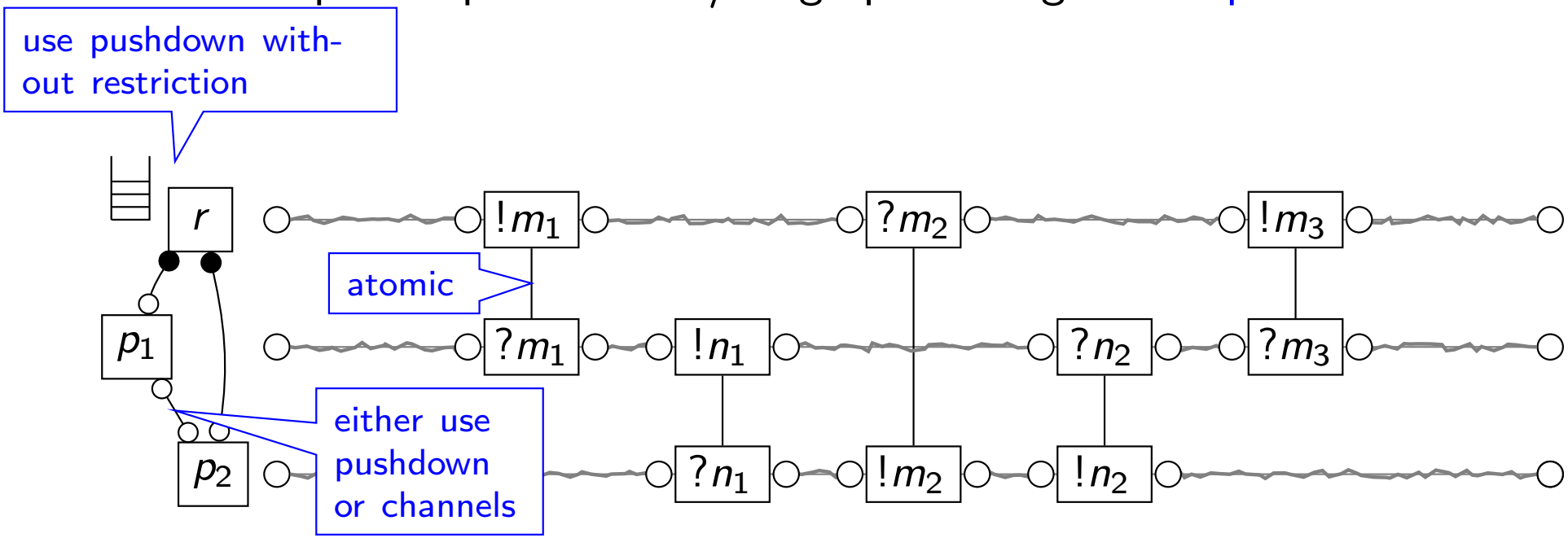

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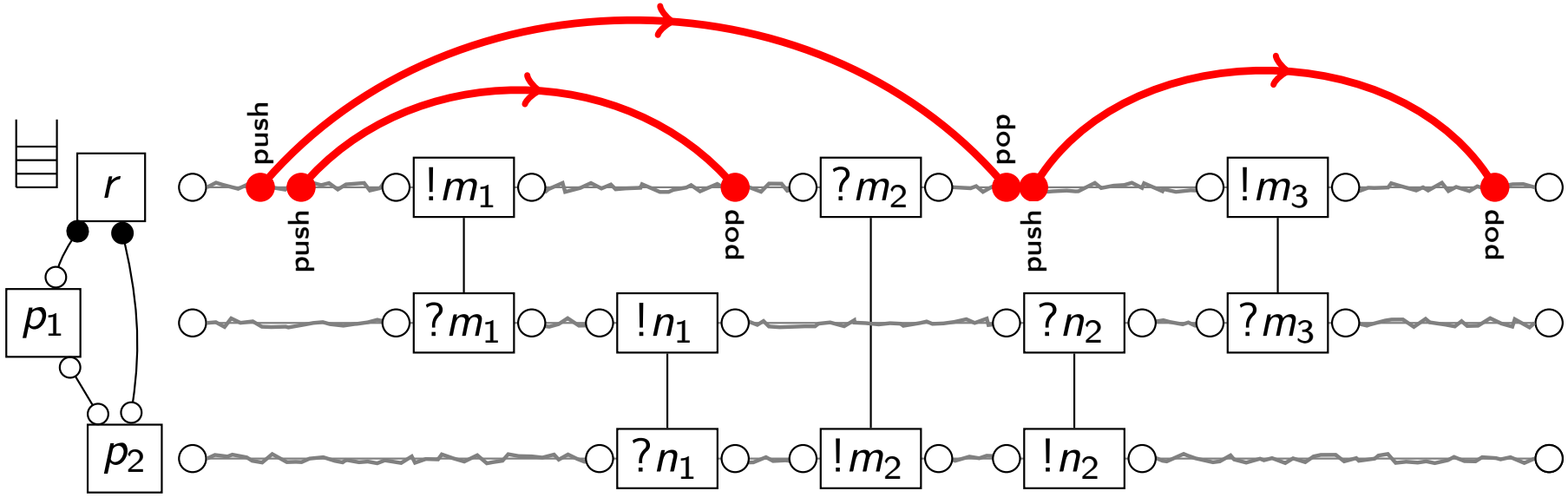

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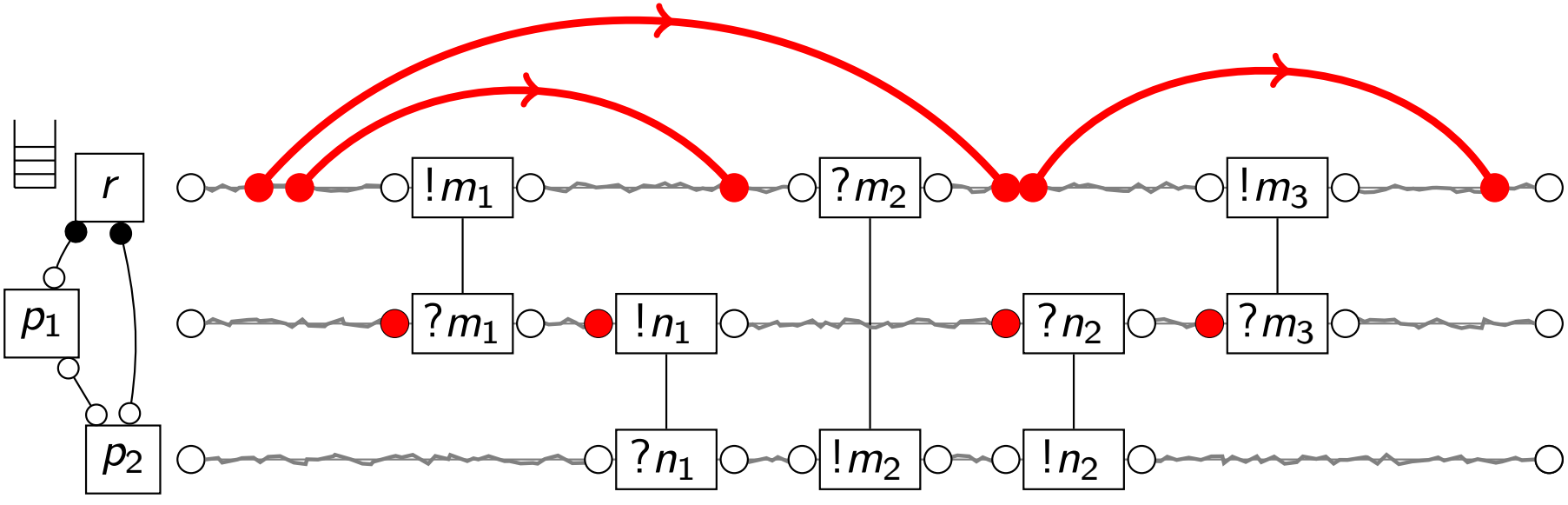

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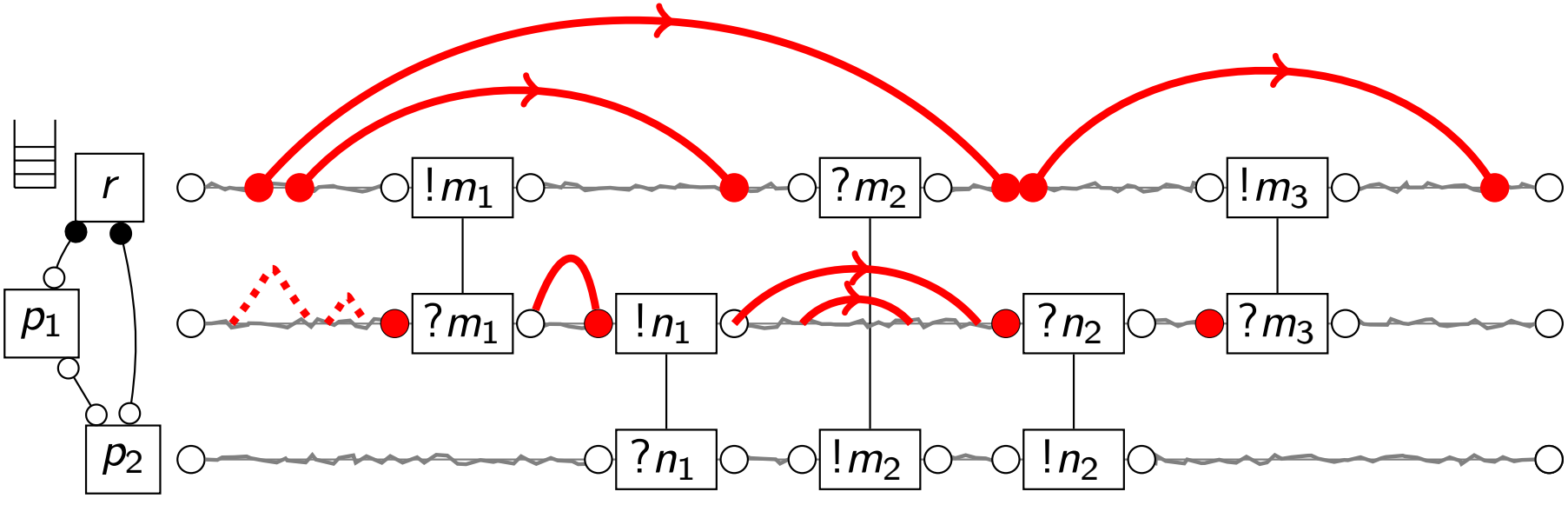

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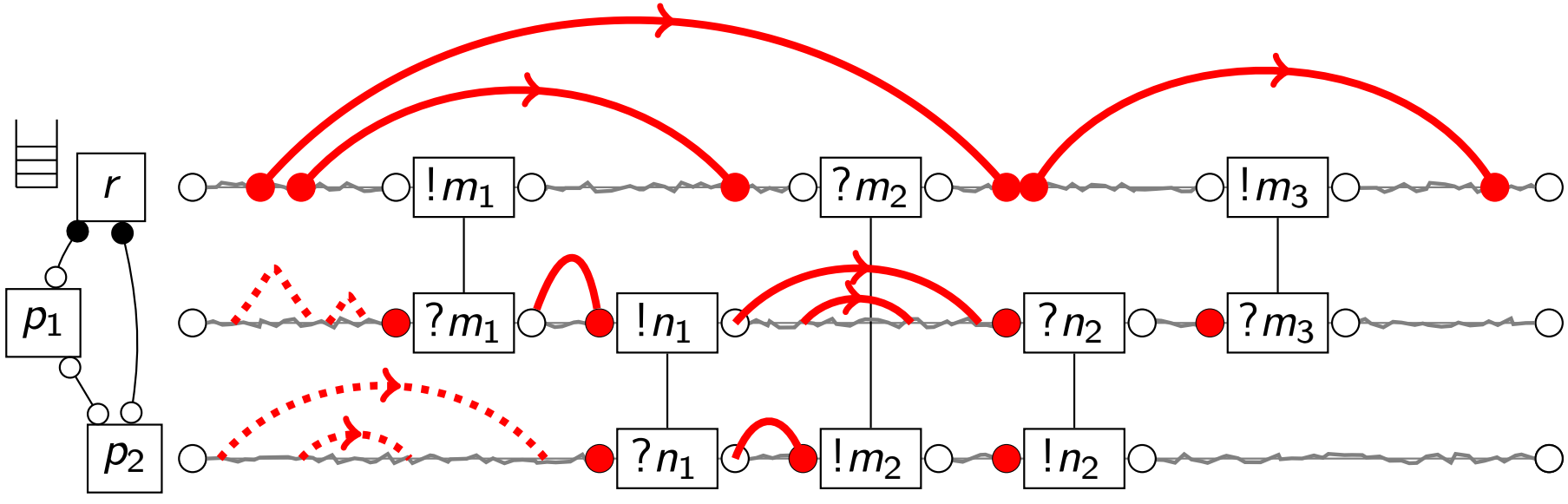

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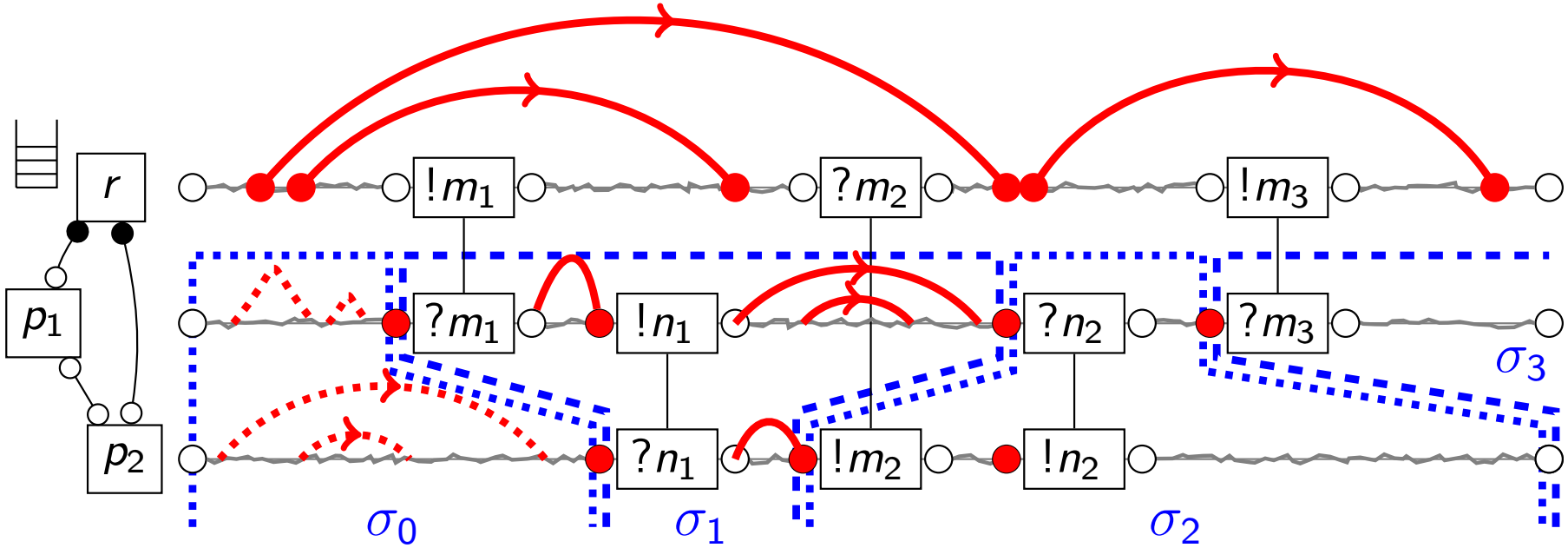


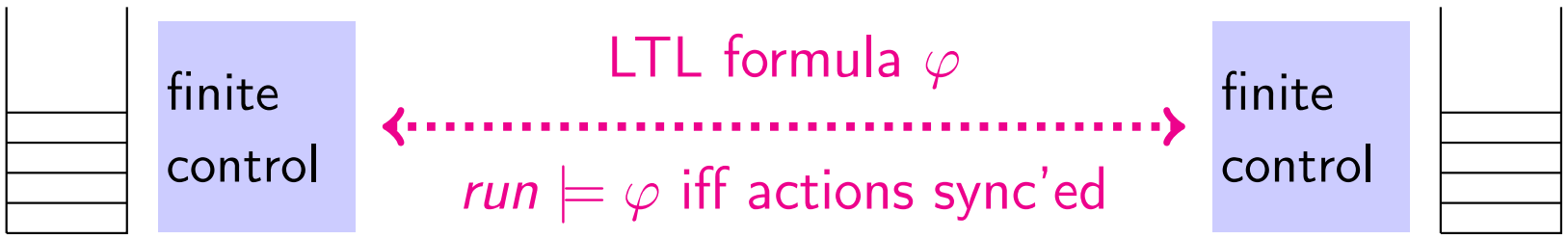
Bounded Ressources and LTL-MC

➪ however extending this result leads directly to:

Theorem:

The LTL model checking question is undecidable for RQCPS that are well-queueing, eager and imbalanced. This already holds for a weak fragment of action LTL.

➪ couple independent events by LTL interpreted on interleaving ↯



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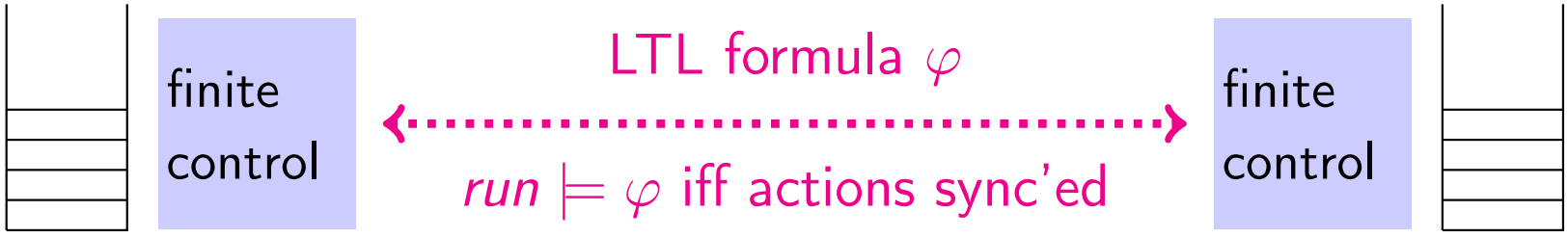
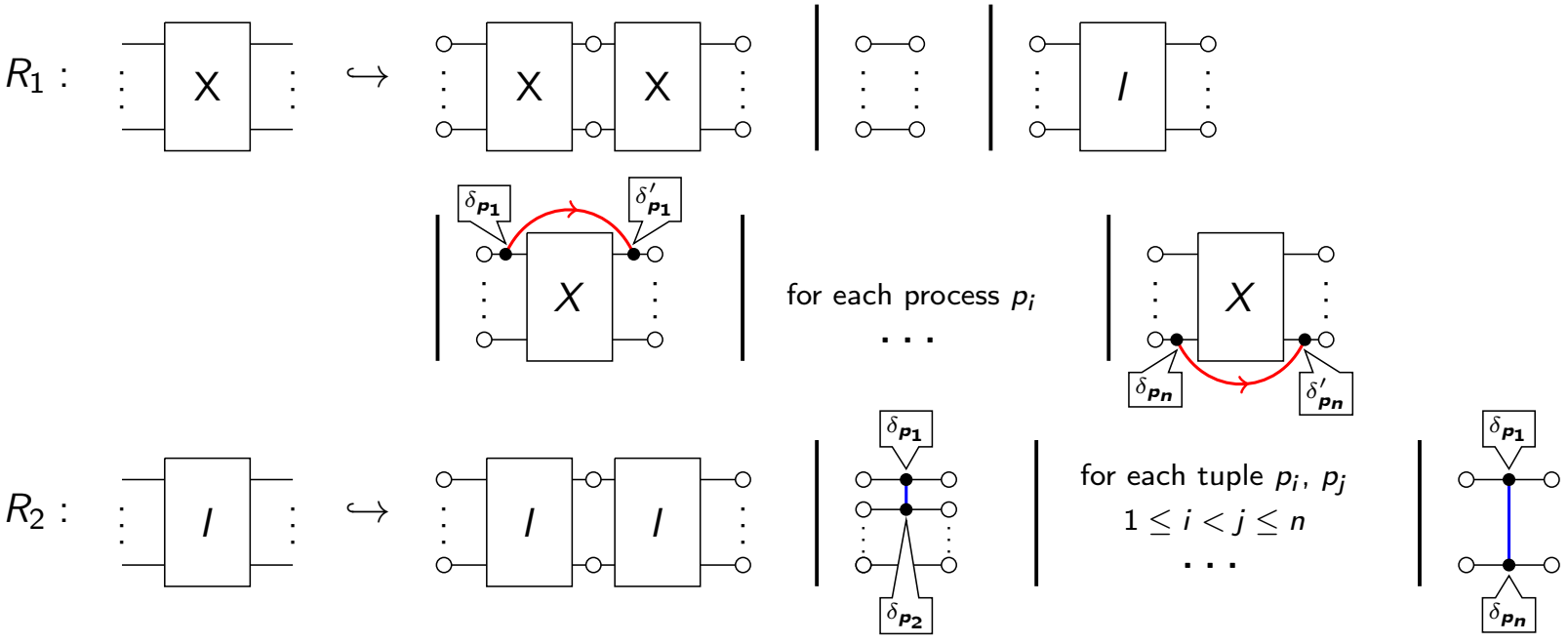


➪ leave interleaving semantics behind, look at rungraphs !

HRG for 1-stack reducible eager iRCPS



- guess "correct" labelling (transition rule $\delta \in \Delta_p$ f.e. process p)
- rulewidth bounded by $\mathcal{O}(\#\text{procs.})$
- correctness of labelling "local": express by MSO formula φ_{label}
- model check in addition: $\varphi' \equiv \varphi \wedge \varphi_{\text{label}}$

MSO Model Checking RCPS

Theorem:

The MSO model checking question for rungraphs of RCPS that are eager, imbalanced and for which all runs can be reordered such that they are "1-stack reducible" is decidable.

- ➪ cannot decide if for a given RCPS all runs can be reordered into a 1-stack one ↯
- ➪ but covers important practical examples ☺
  (e.g., master-worker protocols etc.)
- ➪ semantic boundedness versus syntactic description ☹

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Treewidth Boundedness

Conjecture:

Bounded treewidth of the underlying rungraphs is the most general form of semantic boundedness condition for which MSO model checking is decidable.

➪ can generalize other boundedness conditions
(e.g., bounded phases for RCPS ✓ , lock graphs ✓ , etc.)

? how to derive algorithms that are usable in practice

? how to derive syntactic/decidable restrictions

? what about other graph boundedness measures
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Related Works

- [Madhusudan et al.] LTL/MSO model checking for MSC-graphs
- [Parlato et al.'08] use tree-decomposition to derive new model checking algorithms (bounded phases...)
- MSO-definable temporal logics [Kuske/Gastin'10] and verification on partial orders of RCPS [Bollig et al.'11]
- partial-order versions of LTL (no consensus)
- novel boundedness conditions for asynchronous systems [LaTorre/Napoli'11] [Bouajjani/Emmi'12] etc.
- `Autowrite` tool [Durand/Courcelle]
- graph-rewriting based transition systems

Final Summary

- leave behind interleaving semantics
- focus rungraphs
- describe generation of rungraphs by graph grammars
- do model checking on rungraphs
- known connection HRG-treewidth-MSO allows to derive decidability of model checking